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Examining the Effects of College Algebra on Students' Mathematical Dispositions

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ABSTRACT

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As Mathematics Educators, we want to help our students not only develop a deep, conceptual understanding of mathematical concepts and processes, but also a positive disposition towards mathematics. Despite the importance of helping students develop a positive mathematical disposition, little research has been conducted examining how students' dispositions develop as they progress through their mathematical studies. In particular, the effects of college algebra on students' mathematical dispositions is not well understood. To examine the influence of college algebra on students' dispositions, students using two different college algebra curriculums were studied at Brigham Young University. Using a mathematical disposition survey, student interviews, and open response surveys, data were gathered about changes in students' dispositions as they progressed through the course. Results suggest that college algebra, on average, does not improve students' mathematical dispositions, and can actually be harmful to students' beliefs about mathematics being sensible and useful, students' beliefs about the importance of hard work and perseverance, and students' self-efficacy beliefs. However, the *Pathways* college algebra course, which was context-based and conceptual in nature, was less harmful than a more traditional college algebra course. These results corroborate other college educators and researchers' perceptions that the content of college algebra needs to be reexamined and changed, in addition to how it is taught.

Keywords: dispositions, mathematics, self-efficacy, perseverance, college algebra

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Chapter 1: Introduction

As Mathematics Educators, we want to help our students not only develop a deep, conceptual understanding of mathematical concepts, but also a positive disposition towards mathematics. This need to develop and cultivate students' mathematical dispositions has been advocated for decades (National Research Council, 2001; NCTM, 1989, 2000). Despite the importance of helping students develop a positive mathematical disposition, little research has been conducted examining how students' dispositions grow and develop, or fail to, as they progress through their mathematical studies (Gresalfi, 2009; Jansen, 2012; McClain & Cobb, 2001; Royster, Harris, & Schoeps, 1999; Smith & Star, 2007). More particularly, our understanding of the specific aspects of mathematics courses that affect and change students' dispositions toward mathematics, either positively or negatively, is very limited. If we want to be able to support students development of productive dispositions, more research needs to be done to better understand how classes affect dispositions.

One mathematics course that could be an important influence on students' mathematical dispositions, and a perfect place to examine how dispositions develop and change, is college algebra, a course that is considered by many college educators to be failing (Gordon, 2008; Herriott, 2006; Owens, 2003; Small, 2006). While college algebra has been traditionally thought of as a precursor to calculus, more and more students are taking college algebra as their very last mathematics course (Small, 2006), and for many, the only mathematics course they will take in college. Thus, college algebra is the last opportunity we have of helping students' mathematical dispositions improve. College algebra needs to not only prepare calculus-bound students for higher mathematics, but more importantly, it needs to help students become more mathematically proficient for success in our advancing world, which includes developing a positive disposition towards mathematics.

In the book *Adding it Up: Helping Children Learn Mathematics,* the National Research Council (2001) define a *productive disposition* as a “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (p. 116). The National Council of Teachers of Mathematics (NCTM) emphasized the importance of students developing productive mathematical dispositions in their *Curriculum and Evaluation Standards* (1989):

> Learning mathematics extends beyond learning concepts, procedures, and their applications. It also includes developing a disposition toward mathematics and seeing mathematics as a powerful way for looking at situations. Disposition refers not simply to attitudes but to a tendency to think and to act in positive ways. (p. 233)

NCTM (1989) goes on to explain that a positive mathematical disposition leads students to value mathematics as a powerful tool that is useful in other disciplines and in solving everyday problems, as well as helping students to become intellectually autonomous as they recognize their own abilities to use mathematics in problem solving. Furthermore, those who have a productive disposition towards mathematics (National Research Council, 2001) understand the importance of effort and perseverance in mathematics, are more likely to endure through setbacks and difficulties when learning mathematics, and are less likely to give up the study of mathematics, but rather relish in its challenge (Dweck, 2006) and pursue further mathematical studies and mathematical careers.

Many colleges and universities have recognized the importance of college algebra, particularly as a gateway to mathematically concentrated studies and careers, and have sought to improve and strengthen this course, with some success in increasing passing rates (Burmeister, Kenney, & Nice, 1996; Dedeo, 2001; González-Muñiz, Klinger, Moosai, & Raviv, 2012), supporting mathematical understanding (Shoaf-Grubbs, 1994), and improving student attitudes towards mathematics (Hagerty, Smith, & Goodwin, 2010; Hodges & Kim, 2013). While the effects of college algebra on students' general attitudes towards mathematics (i.e. "I like mathematics," or, "I do not like mathematics") has been examined by researchers, there is much more to dispositions than whether or not a student likes or dislikes mathematics, including the aspects of a *productive disposition* mentioned above. More research needs to be conducted using the lens of mathematical dispositions, examining how college algebra might affect students' dispositions towards mathematics.

The purpose of this study is to better understand the effects of college algebra (both a traditional course and a redesigned, context-based course) on the mathematical dispositions of students, particularly the specific aspects of the course that might be having an effect on students' dispositions. With a better understanding of the ways college algebra might be affecting students' mathematical dispositions, we can then better explore ways of designing and implementing courses that should be more likely to improve those dispositions.

Chapter 2: Background

Theoretical Framework

While there is agreement within the mathematics education community on the importance of developing students' mathematical dispositions (National Research Council, 2001; NCTM, 1989, 2000), what researchers have meant by "dispositions" has varied. Some have used the term "dispositions" to refer to students' general attitudes or beliefs towards mathematics (Gutstein, 2003; Kuhs & Ball, 1986; Törner, 2002). Others have recognized that dispositions should not just refer to how students generally feel about mathematics, but also to beliefs about the importance of making mathematical connections, and the importance of hard work and diligence to mathematical success (Beyers, 2011). Still, others have added on further ideas to the term "disposition":

- McIntosh (1997) defined dispositions as "one's usual mood; temperament, a habitual inclination, [or] tendency" (p. 95), but also added that dispositions include attitudes, persistence, confidence, and cooperative skills.
- Gresalfi and Cobb (2006) used the term disposition as encompassing "ideas about, values of, and ways of participating with a discipline that students develop in a particular class and as they move from one class to another" (p. 50).
- Gresalfi (2009) later defined dispositions as "ways of being in the world that involve ideas about, perspectives on, and engagement with information that can be seen both in moments of interaction and in more enduring patterns over time" (p. 329).
- NCTM (1989) in the *Curriculum and Evaluation Standards for School Mathematics*, stated that "disposition refers not simply to attitudes but to a tendency to think and to act in positive ways" (p. 233).

- Royster et al. (1999) used the NCTM (1989) definition as a basis for their conceptualization of dispositions, also adding that confidence, perseverance, and interest are all components of students' mathematical dispositions.

Within all of these definitions are common themes, such as beliefs, attitudes, actions, perseverance, and confidence, but there is enough variation that makes it necessary for me to define clearly what I mean by "disposition" before I am able to examine how college algebra effects students' mathematical dispositions.

The definition of mathematical disposition I chose to use for my study is based off of an oft cited definition given by the National Research Council (2001) in their book *Adding it Up: Helping Children Learn Mathematics*. The book explains efforts made by a select national committee to define what successful mathematics learning entails. They chose the term *mathematical proficiency* to capture what is necessary for anyone to be successful in learning and using mathematics. They then named five strands of this mathematical proficiency:

- *conceptual understanding*--comprehension of mathematical concepts, operations, and relations
- *procedural fluency*--skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*--ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*--capacity for logical thought, reflection, explanation, and justification
- *productive disposition*--habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (p. 116).

In examining the definition the National Research Council (2001) gave for a productive disposition, I found that the definition encompassed what I believe a mathematical disposition entails: a set of beliefs which influences a person's mathematical confidence, their willingness to persevere in the face of obstacles in solving mathematical problems, and their views about the understandability, usefulness, and worthwhileness of mathematics. These beliefs then guide and influence how students act with mathematics. However, the components of a productive disposition were not well defined in *Adding it Up*. Furthermore, the National Research Council (2001) only defined a *productive* disposition, not mathematical dispositions in general. In fact, when talking about mathematical dispositions, mathematics educators seem to have a tendency of talking about them only positively, or solely focusing on improving dispositions as if there is no way to *harm* a students' mathematical disposition (National Research Council, 2001; NCTM, 1989). Thus, for my theoretical framework, I needed to further examine and expound upon the elements that make up a productive disposition, and then use these elements in a way that allows me to describe any students' disposition towards mathematics, not just those that are *productive*.

For this thesis, I define a students' mathematical disposition as consisting of three main components:

1. *The Self-Efficacy Component*—A student's belief about their capability of using mathematics effectively and successfully.
2. *The Math as Sensible Component*—A student's beliefs about the degree to which mathematics is sensible, useful, and worthwhile.
3. *The Perseverance Component*—A student's beliefs about the importance of persistence and diligence in determining success in solving mathematical problems.

In particular, the three components can be thought of as constituting three continuums, as shown in Figure 1.

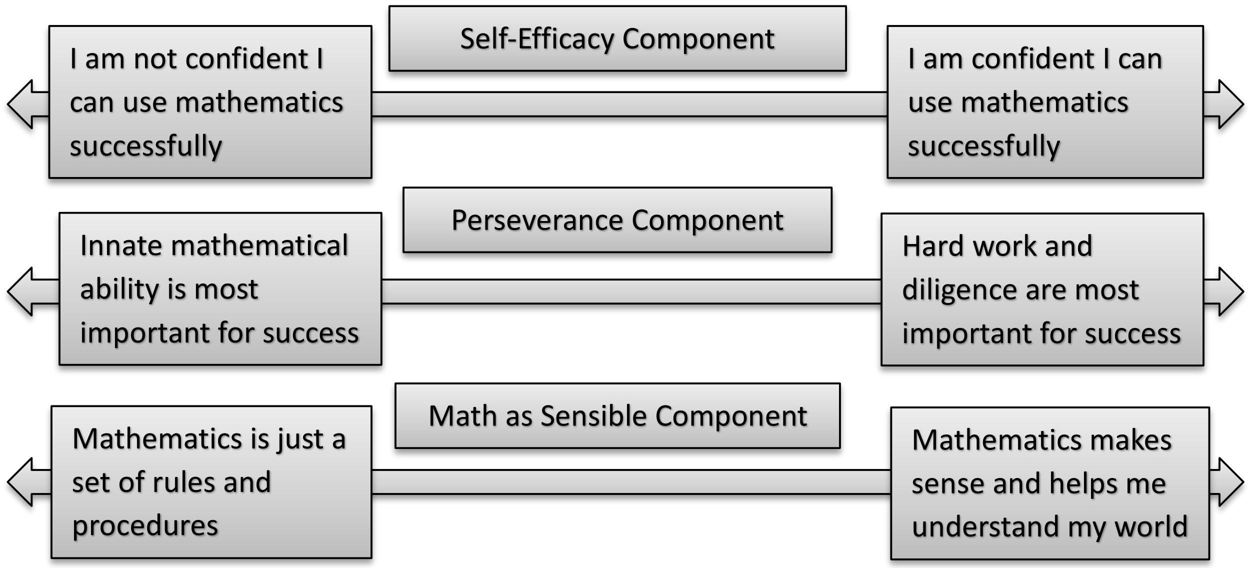

Figure 1. Continuums of components of mathematical dispositions.

On the Self-Efficacy continuum, students can range from feeling like they cannot ever use mathematics successfully to solve future problems, to being confident that they could use mathematics successfully. On the Math as Sensible continuum, students can range from viewing mathematics as a set of rules and procedures that should be memorized simply to pass mathematics courses, to seeing mathematics as a sensible, logical subject that is highly useful for understanding the world around us. On the Perseverance continuum, students can range from a belief that innate mathematical ability is what is most important to be successful in mathematics, to a belief that hard work and diligence are what is most important for success.

We can assess a students' mathematical disposition by locating where they would fall on each of the three continuums. For example, a student might believe they will never be successful in using mathematics, that mathematics is just a game of memorization that is completely useless, and that the only way to be successful in mathematics is to be a "math person." This student would be found on the far left of each of the continuums described. However, a student does not have to be on one side for all three continuums. We can conceivably think of a student who believes they will definitely be successful in using mathematics in the future, but still holds a belief that mathematics is mostly about memorizing rules and procedures, and feels that innate mathematical ability is crucial to success; this student would be on the far right of the self-efficacy continuum, but on the left side of the other two. Thus, a student's beliefs could place them anywhere on each of the individual continuums.

Students who have a *productive* disposition, as defined by the National Research Council (2001), would fall on the right side of each of the three continuums. The beliefs that exist on the right side of the continuums can be considered more productive than those beliefs on the left, and in order to understand why this is so, I will expound and expand upon each of these components in the following sections.

The Self-Efficacy component. Bandura (1986) defined self-efficacy beliefs as "people's judgments of their capabilities to organize and execute courses of action required to attain designated types of performances" (p. 391). These personal judgments are one of the most central mechanisms of agency, influencing how people feel, think, motivate themselves, and behave (Bandura, 1993; Schunk & Pajares, 2009). Self-efficacy beliefs bring about these influences through four major processes, namely cognitive, motivational, affective, and selection processes (Bandura, 1992), which I will further explain.

The effects of self-efficacy on cognitive processes are manifest in a variety of ways, but one of the biggest effects is upon performance. "There is a marked difference between possessing knowledge and skills and being able to use them well under taxing conditions … Hence, a person with the same knowledge and skills may perform poorly, adequately, or extraordinarily depending on fluctuations in self-efficacy thinking" (Bandura, 1993, p. 119). A major goal of mathematics education is to help students use mathematical tools successfully, and without improving students' self-efficacy beliefs, many may struggle to do so, due to their preconceived beliefs about their abilities to be successful in mathematics. For example, Pajares and Miller (1995) found that students' beliefs about their own problem solving abilities had a significant effect upon problem-solving performance, with higher self-efficacy about their problem solving leading to better performance in problem solving tasks, and lower self-efficacy about problem solving leading to lower performance. In fact, students with the same mathematical skills can vary vastly in their ability to use those skills based only on their self-efficacy beliefs (Schunk & Pajares, 2009).

Bandura (1993) also explains that self-efficacy beliefs influence motivation in several ways. First, individuals who feel highly efficacious attribute failures to lack of effort, whereas those with low self-efficacy attribute failures to inability or low ability. Second, self-efficacy beliefs largely influence the goals that people set for themselves. Those with high self-efficacy beliefs set challenging goals for themselves, believing that they have the abilities to accomplish it, while those who have low self-efficacy about a specific domain will set low goals (or no goals at all) feeling that they do not have strong abilities to accomplish the goal. Third, and related to setting goals, self-efficacy beliefs have a strong impact on the amount of effort people expend in order to accomplish their goal (see also Schunk & Pajares, 2009). Finally, self-efficacy beliefs greatly affect how long people persevere towards accomplishing their goals in the face of difficulties, and how people react to those difficulties. Thus, a students' mathematics-related self-efficacy can have a profound effect upon the mathematical goals they set for themselves, how long they pursue those goals, and what the students will do in the face of difficulty, either persevering, or simply giving up because of "lack of ability."

In the affective domain, self-efficacy beliefs not only affect motivation, but they also affect how much stress and depression people experience in difficult situations (Bandura, 1993). Those with low self-efficacy beliefs have high anxiety arousal, dwell on their coping deficiencies, view many aspects of their environment as fraught with threats, magnify the severity of possible threats, and worry about things that hardly ever happen. Through all of this anxiety and worrying they impair their level of functioning, and thus perform more poorly than they would normally. This is all too evident in the mathematical domain. Many Americans feel extremely anxious about mathematics, and almost wear it proudly as a badge ("I was never good at math"). Many feel that mathematics is "out to get them" as they go through school, and feel that they cannot control how well they understand mathematics, or perform well in math classes. This "math anxiety" appears to be a learned behavior (Turner & Meyer, 2009), which is largely influenced by the experiences students have with mathematics in school.

Selection processes, or the choices and decisions we make, are also affected by self-efficacy beliefs. They "shape the course lives take by influencing choice of activities and environments. People avoid activities and situations they believe exceed their coping capabilities. But they readily undertake challenging activities and select situations they judge themselves capable of handling" (Bandura, 1993, p. 135). One example of these selection processes in mathematics is whether or not students choose to take higher level mathematics courses. Those who have high self-efficacy in mathematics are usually the ones who go on to take courses such as Calculus or Differential Equations. Another example, in relation to this, is career choice. A person will most likely avoid mathematically intensive fields if they do not believe that they are capable of understanding and using the mathematics involved in that particular field.

Self-efficacy, “people’s beliefs about their capabilities to exercise control over their own level of functioning and over events that affect their lives” (Bandura, 1993), can thus be seen as a set of beliefs that strongly affect our thoughts, feelings, and behavior. More specifically, mathematics-specific self-efficacy is the set of beliefs people hold about their capabilities to function in mathematics classes and in mathematical problem solving, affecting how people think about, feel about, and act in regards to mathematics. Furthermore, those who hold higher self-efficacy beliefs have a more productive disposition because they are better able to cognitively perform well, are more motivated to persist in the face of difficulties, have less math anxiety, and are more likely to pursue further mathematical studies and careers.

The Math as Sensible component (Beliefs about the degree to which mathematics is sensible, useful, and worthwhile). Skemp (1978/2006), in one of his most well-known articles, defined two different types of mathematical understanding: relational understanding, and instrumental understanding. Relational understanding is knowing both what to do, and why it works. Instrumental understanding, on the other hand, is the possession of rules and procedures that allow someone to get the right answer without understanding the reasons behind them. Skemp further explained that, although instrumental mathematics has its advantages (it can be easier to understand, has more immediate rewards, and allows you to get the right answer more quickly), the advantages of relational mathematics are much greater. These advantages include understanding mathematical concepts in ways that (1) make it more adaptable to new tasks, (2) make it easier to remember, (3) make it an intrinsically rewarding goal in itself, and (4) cause students to seek out their own learning opportunities in mathematics. A student with a productive mathematical disposition is a student who believes mathematics can be understood relationally, seeing mathematics as logical, sensible, useful, and worthwhile. These students believe that mathematical concepts should all make sense. They strive to understand not just the “what” and “how” of mathematics, but also the “why,” seeking to make connections in all mathematics they learn, because they believe that all mathematical ideas should make sense.

Sadly, for most students, mathematics is not seen this way. As Kaput (1989) once eloquently explained:

> … the experience of meaningfulness of the mathematics [is neglected] … Few now deny that school mathematics as experienced by most students is compartmentalized into meaningless pieces that are isolated from one another and from the students’ wider world. Symbols are manipulated without regard to the meanings that might be carried, either by referents of the symbols or by actions on them. Theorems are ‘proved’ without the slightest attempt to generate the statement to be proved or to justify the need for proof. This *experienced meaninglessness* of school mathematics devastates the motivation to learn or use mathematics and is entirely incompatible with a view of mathematics as a tool of personal insight and problem solving. This core problem of alienation is compounded by the difficulties inherent in dealing with formal symbols (e.g. algebra, isolated from other knowledge). (pp. 99-100)

Thus, more often than not, students begin to see mathematics as a list of memorized rules and procedures that have little to no meaning.

Boaler and Greeno (2000) provided an excellent example of this meaningfulness versus meaninglessness in mathematics classes through their study of 48 students enrolled in advanced placement (AP) calculus. In two classes, students were encouraged to work collaboratively and to discuss concepts and meanings with each other. Students in these classes found mathematics meaningful, expressing that they were able to “get into” the mathematics, and understand the concepts they were learning. In contrast, students in the other four, more traditional classes perceived their learning as simply memorizing procedures and repeating them to get the "right" answer. Although some students found security in these "didactic" classrooms, most were frustrated with only being presented with the procedures. As one student said, "We knew HOW to do it. But we didn't know WHY we were doing it. … And I think that's what I really struggle with is--I can get the right answer, I just don't understand why" (p. 184). Interestingly enough, many fewer students in the didactic classes expressed a desire to continue studying mathematics, due to the concepts of mathematics being taught as a large group of procedures to memorize with no rhyme or reason to them.

The importance of seeing mathematics as sensible was stressed by the National Council of Teachers of Mathematics (NCTM) in the Learning Principle section of the Principles and Standards for School Mathematics (2000): "Students will be served well by school mathematics programs that enhance their natural desire to understand what they are asked to learn" (p. 21). NCTM (2000) continues by stating the importance of giving students opportunities to develop conceptual understanding and helping them develop a belief that mathematics is a subject that can be understood. By actively engaging students in tasks and experiences designed to deepen and connect their mathematical knowledge, students are encouraged to become autonomous, lifelong learners, seek understanding, persevere through difficulties when they arise, and see mathematics as personally useful. This will then increase their belief that mathematics is sensible, useful, and worthwhile.

We need to be careful, however, and further define what we mean by students having a belief that mathematics is useful and worthwhile. Even though "students appear to think mathematics is useful for everyday problems or important to society in general, it is not clear that they think it is important for them as individuals to know a lot of mathematics" (National Research Council, 2001, p. 141). This leads us to examine two ways in which learning (including mathematical learning) can be seen as valuable. The first is an extrinsic value, known as exchange value, wherein "the primary aim is to exchange one's education for something more substantial--namely a job, which will provide the holder with a comfortable standard of living, financial security, social power, and cultural prestige" (Labaree, 1997, p. 31). Often this is the argument many teachers give for learning mathematics: "If you do well in this class, you can go to college, and get a better job." While this is one way to value the learning of mathematics, it is arguably not a powerful motivator for all students. The second value of learning and education is intrinsic, known as use value: "the citizen and the taxpayer (or employer) place value on education because they consider the content of what is learned there to be intrinsically useful" (Labaree, 1997, p. 31). Students with productive dispositions toward mathematics, in seeing mathematics as useful and worthwhile, might view mathematics learning as having exchange value, but are more importantly able to see mathematics as having use value, being directly applicable to their own lives, and personally useful.

When students see mathematics as rewarding, useful, and sensible, it is more likely that they will genuinely like mathematics, and have a much greater desire to continue to study math (Turner & Meyer, 2009). The goal for mathematics educators, therefore, is to help students develop a habitual inclination of seeking meaning behind mathematical concepts, making mathematical connections, and looking for mathematics' intrinsic value.

The Perseverance component. Problem solving is not only an integral part of doing mathematics (Romberg, 1994), but also a major means of learning math (NCTM, 2000). Additionally, becoming a good problem solver can be a huge benefit in everyday life. "By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom" (NCTM, 2000, p. 52). In life outside of mathematics classrooms, students will be confronted with difficult situations that will require these problem solving skills, including the critically important ability and willingness to persevere and persist even when obstacles and challenges arise. This willingness follows naturally from a belief that the most important factor leading to success in mathematics is hard work and perseverance.

Others have mentioned the importance of a belief that persistence and diligence lead to success in true problem solving. Fennema and Peterson (1985) identified attributes needed for accomplishing complex tasks called Autonomous Learning Behaviors (ALB). These attributes and skills include being able to work independently, persist, choose, and succeed at such tasks. The expert problem solver and teacher George Polya, in his book "How to Solve It" (1945), also specifically mentions persistence as an important aspect of problem solving:

> It would be a mistake to think that solving problems is a purely 'intellectual' affair; determination and emotions play an important role. … to solve a serious scientific problem, will power is needed that can outlast years of toil and bitter disappointments (p. 93).

Thus, perseverance is crucial for success in problem solving and in using mathematics, and hence a belief in the importance of hard work and diligence is key.

Unfortunately, many students never get the opportunity to truly problem solve, and, as a result, never learn the importance of perseverance, or never develop a belief that hard work and diligence are keys to problem solving. Instead, students are bombarded with bite-sized exercises aimed at helping the student achieve subject mastery (Schoenfeld, 1989). These exercises usually take no longer than ten minutes to solve. When the exercise takes longer than that, many students become frustrated or even give up, sometimes believing that the problem is unsolvable. Without opportunities to see the fruits of success from persevering in the face of difficulties, students are probably more likely to believe that innate ability in mathematics is more important than hard work and persistence.

Self-efficacy beliefs also have a large effect on persistence in problem solving, and students' beliefs about its importance. As mentioned previously in this paper, the amount of time and effort that one spends in trying to accomplish a goal (such as solving a problem) is largely influenced by one's own efficacy beliefs (Bandura, 1993). A student with high self-efficacy will spend much more time, and is much more willing to persevere in the face of difficulties, than a student with low self-efficacy. Students with low self-efficacy beliefs are also much more likely to give up quickly when faced with complications while solving problems (Bandura, 1993; Schunk & Pajares, 2009). In other words, a students' beliefs about how successful they will be solving problems, specifically mathematical problems, affects their beliefs about how important it is for them to persevere in the face of difficulties. Self-efficacy further affects persistence through people's views about the extent to which their environment is controllable.

> People who are plagued by self-doubts anticipate the futility of efforts to modify their life situation … But those who have a firm belief in their efficacy, through ingenuity and perseverance, figure out ways of exercising some control, even in environments containing limited opportunities and many constraints (Bandura, 1993, p. 125).

Therefore, those with high self-efficacy beliefs in mathematics, such as those with a productive mathematical disposition, are more likely to believe that they can persevere and work hard to be successful in any mathematics class, or in solving any mathematical problem.

These ideas about perceived controllability are heavily related to attribution theory. Attribution theory explains student motivation and perseverance in terms of students' perceptions of the reasons for their successes and failures rather than as simply a drive or a trait (Kloosterman, 1988). Diener and Dweck (1978) more particularly found a marked difference in performance, diligence, and persistence between students who were helpless (perceived themselves as having an inability to overcome failure) to mastery-oriented students (students who attribute failure to lack of effort). They found that helpless students frequently attributed their failure to lack of ability, but mastery-oriented students did not explicitly attribute their failure to anything and rather focused on finding a remedy for the failure, through further effort and persistence. Later, Dweck (2006), in her book *Mindset: The New Psychology of Success*, gave names to these two different mindsets of thinking. Those with a *fixed mindset* believe that one's abilities and talents are fixed, and nothing can be done to improve your natural talent. You are either good at something automatically and will excel, or, through lots and lots of hard work, you may be moderately able to succeed. If something requires effort, than you are not very good at it. On the other hand, those with a *growth mindset* believe that it is our effort and perseverance that make accomplishment possible and worthwhile; they "may appreciate endowment, but they admire effort, for no matter what your ability is, effort is what ignites that ability and turns it into accomplishment" (Dweck, 2006, p. 53). Thus, those with a productive disposition towards mathematics have a *growth mindset*, believing that they can be successful in mathematics through hard work and diligence, no matter what their natural abilities are.

A belief in the importance of diligence in mathematical problem solving is further affected by the value that students place upon mathematics, and whether students see math as useful and worthwhile. Even students who feel highly efficacious in a given domain (such as mathematics) may not necessarily pursue further course work and study in that domain if they believe that those courses will not help them reach personal goals (Schunk & Pajares, 2009). Thus, if students feel that mathematical problem solving will not be useful in their future, then they may not persevere or persist for very long when mathematical problems become difficult, or see perseverance as important, even if the student is highly capable of solving the problem.

Although the Self-Efficacy Component and the Math as Sensible Component do have influence on students' beliefs about the importance of hard work and perseverance in mathematics and mathematical problem solving, it is conceivable that a student could have a high level of self-efficacy, and believe that mathematics is sensible and personally useful, but still very strongly believe that success in mathematics comes mostly from innate ability. Thus, the Perseverance component still needs to be looked at separately within a students' mathematical disposition. Furthermore, those who firmly believe that perseverance is key for mathematical success, such as those with a productive disposition, when inevitably encountering difficulties with mathematics, are more likely to persist and endure through hardships and be successful in future mathematical studies and careers.

Summary of theoretical framework. Using the definition of a *productive disposition* given by the National Research Council (2001) as a basis, I have defined a student's mathematical disposition as consisting of three components, namely their self-efficacy beliefs, their beliefs about the sensibleness, worthwhileness, and usefulness of mathematics, and their beliefs about the importance of perseverance to mathematical success. Although these components of a students' mathematical disposition are interrelated, and can have influence on one another, they are conceptually distinct. Thus, it is important to individually study each of these three beliefs and examine how they grow and develop, resulting in a students' overall disposition towards mathematics. Moreover, these components of disposition can each be seen as existing on a continuum with beliefs at one end being more likely to lead to mathematical proficiency. Thus, students who believe (1) they can be successful in using mathematics in the future, (2) that mathematics is personally useful and makes sense, and (3) that hard work and effort are most important for mathematical success, have a *productive* disposition and have a higher likelihood of pursuance and success in mathematical studies, and personally seeing its usefulness in their chosen career and life in general. This definition provides a way to examine how a mathematics course, specifically college algebra, effects students' mathematical dispositions.

Literature Review

In this literature review, I will be making an argument that our understanding of how students' mathematical dispositions grow and develop is far from being complete, and, more specifically, our understanding of the ways college algebra might affect students' dispositions is very limited. I will first give an overview of what research says are ways that self-efficacy beliefs, beliefs about mathematics being sensible, and beliefs about the importance of perseverance can be influenced and improved. Then, I will present the research others have done to look at mathematical dispositions, and show that there are still gaps in our understanding of how college algebra and the specific aspects of the course might have an influence on students' mathematical dispositions. Finally, I will set forth the research questions for this thesis.

Developing a productive disposition. By having earlier defined the elements that make up a students' mathematical disposition, we can now examine what research can tell us about ways in which a *productive* disposition might be cultivated and developed. To do so, we have to consider how the three main components of a student's disposition towards mathematics (the

Self-Efficacy Component, the Math as Sensible Component, and the Perseverance Component) can be affected and improved.

Self-efficacy beliefs, and more specifically mathematics specific self-efficacy beliefs, can be influenced by the outcome of behaviors as well as input from the environment (Schunk & Pajares, 2009). Some examples of these influences are: students' interpretation of their actual performances, including the judgments they make about their capabilities in comparison to others' performances (Schunk & Pajares, 2009); students' receiving social persuasions (e.g. "I know you can do it") from others (Bandura, 1997); and students' physiological and emotional states such as stress and anxiety (Bandura, 1997). These influences begin within the family, but as children get older, peers and education become increasingly influential (Schunk & Pajares, 2009).

Pajares and Schunk (2001) explained ways in which a teacher and the classroom structure can positively influence student self-efficacy beliefs. First, teachers need to help students develop positive self-regulatory habits. These habits not only include finishing assignments by deadlines, studying in a place without distractions, organizing time and schoolwork, and accessing appropriate resources when needs arise, but they also include the habit of thinking of oneself as efficacious. "Beliefs of personal competence and of self-worth ultimately become habits of thinking that are developed like any habit of conduct, and teachers are influential in helping students develop the habitual self-beliefs that will serve them throughout their lives" (p. 255). Second, since children learn from the actions of models, the teacher needs to effectively model practices in ways that students can connect with, as well as judiciously select peers for classroom models that students feel intellectually comparable with. Third, the classroom structure should lower the competitive orientation so prevalent in traditional classrooms in order to increase student confidence and perceptions of self-worth. Fourth, teachers' own self-efficacy beliefs about their teaching have a large influence on the development of students' self-efficacy beliefs implying the need for teachers to work on improving their own confidence about their teaching abilities. Fifth, and lastly, rather than having students participate in cheap self-esteem programs, gimmicks, and kits, students must be allowed to engage in authentic mastery experiences, thereby raising competence *and* confidence.

Competence in mathematics, and more specifically the ability of students believing that mathematics is rewarding, sensible, and useful, is best developed through participation in classrooms that focus on relational understanding, rather than procedural or instrumental understanding (Skemp, 1978/2006). NCTM's *Principles and Standards* (2000) help explain that a classroom focused on relational mathematics has "a coherent curriculum [in which] mathematical ideas are linked to and build on one another so that students' understanding and knowledge deepens and their ability to apply mathematics expands" (p. 14). NCTM (2000) later adds that the teacher must give students the opportunity to actively engage in tasks and experiences designed to deepen and connect their knowledge, as well as organize a classroom where students interact, propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills. In addition, Brown, Collins, and Duguid (1989) argue that teachers need to act as practitioners and use the tools of the domain (in our case, mathematics) in wrestling with problems of the world, and help their students to do the same. Thus, students need to work with mathematics in real-world situations and contexts so they can witness the usefulness and necessity of learning and understanding mathematics. Without these experiences, students will have a difficult time seeing the relevance of mathematics to their own life or the problems they face.

How do students learn to persevere in problem solving, and develop a belief that hard work and diligence are most important for success in mathematics courses and in problem solving, specifically mathematical problem solving? First, students have to be given the opportunity to engage in true problem solving; "... you learn to do problems by doing them" (Polya, 1945, p. 5). If the solution method is always obvious or prescribed, persistence never becomes important or valued. Charles and Lester (1982) mention three aspects of a true problem: "1. The person confronting it wants or needs to find a solution, 2. The person has no readily available procedure for finding the solution, [and] 3. The person must make an attempt to find a solution" (p. 5). True problem solving (in mathematics classrooms and in real world applications) thus takes much longer than a couple of minutes to accomplish, because there is no prescribed method to immediately carry out in order to solve the problem. When students engage in solving these problems, they begin to see the value in persevering and pushing through difficulties. Second, students need to be encouraged to work hard and persist in solving those problems, even when students feel like they are at an impasse. This can be done through the use of carefully crafted questions asked by the teacher, such as suggested by Polya (1945). Lester (2013) gave his beliefs about how a teacher can help students learn perseverance in problem solving:

> ...a proficient mathematics teacher must be skillful at -- (1) designing and selecting appropriate tasks for instruction, (2) making sense of and taking appropriate actions after listening to and observing students as they work on a task, (3) keeping tasks appropriately problematic for students [allowing them to struggle and find a need to persist and persevere], (4) paying attention to and being familiar with the methods students use to solve problems, (5) being able to take the appropriate action (or say the right thing) at the

> right time, [and] (6) creating a classroom atmosphere that is conducive to exploring and sharing (p. 262).

Third, students need to have good experiences with being diligent in problem solving, and partaking of the intrinsic reward of pushing through difficulties to reach a solution that is understandable and self-discovered, rather than simply given by the teacher. Fourth, teachers have to help students see, and constantly remind students, that the way to success in mathematics is through consistent, perseverant effort, not through innate ability. In order to do this, teachers must have a strong belief that all students can succeed, and organize their classroom accordingly (Gresalfi & Cobb, 2006). Fifth, teachers must create a classroom environment that cultivates the *growth mindset* within students (Dweck, 2006). All of these are what NCTM (2014) refers to as productive struggle:

> Effective mathematics teaching supports students in struggling productively as they learn mathematics. Such instruction embraces a view of students' struggles as opportunities for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions (p. 48).

This productive struggle helps students to see themselves as capable of finding solutions to difficult problems through hard work and diligence, making it more likely that they will persevere in the face of mathematical challenges in the future.

In summary, mathematics classrooms and teachers that promote the development of positive self-efficacy beliefs, a coherent curriculum in which all mathematical ideas are connected and build on one another, and a wide variety of problem solving opportunities where students engage in productive struggle (NCTM, 2014) and see value and accomplishment in persevering through difficult tasks, should be able to improve students' dispositions towards mathematics, and help them become more productive. However, this research only gives us some idea about elements of a mathematics classroom that can have an effect on students' dispositions towards mathematics. Arguably, there is much more that goes on in a mathematics course and classroom, specifically in college algebra, that could have an effect on students' dispositions, for good or for ill. More research needs to be conducted to identify what these influences might be, and how they affect the mathematical disposition of students.

Research examining students' mathematical dispositions. While many researchers have studied student dispositions, few have specifically examined how dispositions develop. Gresalfi (2009) studied two aspects of what she defined as a students' disposition towards mathematics, namely the ways in which students worked with the mathematical content and made mathematical connections, and the ways in which students worked with each other, and how these dispositions changed throughout an eighth-grade mathematics course. By investigating the dispositions of four students as they engaged in mathematics and worked together in groups, she found that student dispositions are not simply inherent, but are rather the result of what happens in moments. Furthermore, she saw how the aggregation of those moments shapes what is expected in the classroom, and the future opportunities offered. Thus, dispositions grow and develop over time, and are not simply a set of solidified beliefs that are static and unchanging.

For a different perspective on students' mathematical dispositions, Smith and Star (2007) located students moving from a junior high or high school using a traditional curriculum to a high school or university using a reform-based curriculum, and vice versa, and interviewed them about their shift into high school or university. A number of results are relevant for my study.

First, the difficulty of a mathematics course and level of education (most evident in the shift from high school to university) had a greater impact (namely in the negative direction) on dispositions than whether the curriculum being used was traditional or reform-based. Second, those students whose dispositions rose participating in a reform-based high-school curriculum liked non-repetitive contextual (story) problems, group work, and understanding the mathematical concepts. Third, those students moving from a reform-based junior-high curriculum into a more traditional high-school mathematics course had both rising and falling dispositions (7 rising; 10 falling), and furthermore gave completely opposite reasons for the rise or fall in their dispositions. The students whose disposition rose welcomed more challenging content, the absence of context, and more individual work; the students whose dispositions fell missed the features of a reform-based classroom they had left behind. Fourth, the students who felt the most deeply affected by the change in how the mathematics class was conducted were those moving from a more traditional high-school class, to a reform-based university course. Fifth, participation in a reform-based classroom does not automatically imply that student dispositions towards mathematics will improve. This study gives us some ideas about other aspects of a mathematics course, such as the level of difficulty of the course, which have an effect on students' mathematical dispositions.

In further trying to understand how students' mathematical dispositions grow and develop, a few studies have specifically studied the development of students' mathematical dispositions as they go through a single mathematics course. McClain and Cobb (2001) studied the development of sociomathematical norms in a First-Grade classroom, including the development of student dispositions, and found that an emphasis of understanding in the classroom, as well as an expectation that students explain and justify their reasoning, helped students develop a positive disposition towards learning mathematics. More recently, Jansen (2012), in studying the effect of small-group work on the mathematical dispositions of sixth-grade mathematics students, looked at aspects of a productive disposition towards mathematics (National Research Council, 2001), particularly students' having intellectual autonomy, a belief that mathematical competence is improvable, a focus on understanding, and a value for collaborating with peers. She found that students were more likely to develop a productive disposition if the teacher transferred responsibility for learning to the students, explicitly requested multiple solution methods, provided scaffolding that did not lower cognitive demand of the task at hand, and focused on helping students learn with understanding.

The above studies notably pointed out specific aspects of a mathematics course, such as an emphasis on conceptual understanding, and transferring responsibility of learning from the teacher to the students, that help to improve students' mathematical dispositions. However, there are many other elements of a mathematics course that students are involved in which could have an effect on dispositions, such as homework, exams, the teacher, and interacting with the textbook. Thus, it cannot be assumed that all of the aspects of mathematics courses which can influence a student's disposition towards mathematics have been identified and studied. Furthermore, these studies only identified elements of mathematics classes that *improve* students' dispositions; less is known about aspects of a mathematics course that might actually be *harmful* to a student's disposition.

In addition to this, few studies have specifically looked at college algebra, and how it might affect students' mathematical dispositions. Hagerty et al. (2010) did mention that they had attempted to give surveys to students assessing their self-efficacy in regards to mathematics while taking a redesigned college algebra course, but they were not able to use these results in their paper due to the authors' changing ideas of self-efficacy, which caused them to redesign the surveys from semester to semester. More promisingly, a recent study by Hodges and Kim (2013) used a motivational design model developed by Keller (1987) called ARCS (attention, relevance, confidence, and satisfaction) to design an online college algebra course aimed at improving student attitudes towards mathematics. The main aspect of this modified course was a motivational video shown to students in their experimental group. Statistically significant results were obtained that showed that the motivational video improved students' attitudes toward mathematics, while the attitudes of those in the control group actually *decreased*. This study showed that there are ways in which college algebra might be able to improve students' attitudes toward mathematics, albeit this was not a course redesigning. It also shows that when nothing is done to improve college algebra courses, students' attitudes towards mathematics can actually drop as they go through the course.

Research Questions

While researchers have been able to find many aspects of mathematics courses that effect the development of students' mathematical dispositions, there are arguably many other elements that have not been identified, such as the structure of the course, homework, and tests, which could be having an influence on students' dispositions, for both good and ill. Additionally, little has been done to examine how students' dispositions towards mathematics are affected by a college algebra course, only looking at improving students' general attitudes towards mathematics, or attempting to study self-efficacy. This study will attempt to remedy some of the gaps in our knowledge about the development of students' mathematical dispositions and elements of mathematics courses that might be influencing their development, specifically within a college algebra course, by answering the following questions:

1. What are the effects of college algebra classes (both a non-traditional, context-based course [*Pathways*], and a traditional course) on students' mathematical dispositions?
2. What aspects of the college algebra experience (*Pathways* and traditional) seem to or might have an effect on students' mathematical dispositions?

Chapter 3: Methods

In this chapter, I will describe the methods I used to answer my research questions. First, I will explain the setting within which the data was gathered. Second, I will explain how participants were selected, and the different instruments that were used to collect the data. Third, I will describe how the data was analyzed to address my research questions.

Setting

My research was conducted at Brigham Young University (BYU), where a unique opportunity existed to examine the mathematical dispositions of students in two different college algebra classes, using two different curriculums. The first course was run by the Mathematics Department, and could be considered a traditional college algebra course. Topics of study included: (1) functions and their graphs, (2) polynomial and rational functions, (3) exponential and logarithmic functions, (4) conic sections, (5) systems of equations and inequalities, (6) sequences and induction, and (7) counting, probability, and the binomial theorem (Lissitz & Samuelsen, 2007). Tests and assignments, for the most part, assessed whether or not students could use the correct procedures given by the teacher to obtain the right answer. Most of the classes were organized so that students attended lecture twice a week for one hour, and also a lab with a Teaching Assistant twice a week for one hour, thus meeting for four hours each week. Lectures had 45-85 students and met in a large lecture hall. The lab was a chance to meet with a smaller group of students, typically around 15-20, in a smaller classroom where the TA helped students with questions the students had from the lecture, taught content that was not covered in the lecture, and helped students work through problems from their homework. However, there was also one evening section consisting of 17 students that some of the students in my study were in which only met twice a week for two hours, solely with the instructor.

This traditional college algebra class had a typical, procedure-based textbook, wherein concepts were taught centered on procedures and tricks for solving specific types of mathematical problems, and a number of examples were given where the students were shown how to use the procedure step-by-step. Homework was completed completely online with a program specifically tied to the textbook. Homework usually consisted of problems where students were required to solve the problem, and then enter their answer into a given space in a specified format. When students struggled to solve a given problem, there were buttons built into the online program that either gave hints, or actually walked the students through the problem step-by-step. These step-by-step walkthroughs used different numbers from the problem given to the students to solve, but this was the only difference. If students followed the step-by-step instructions, they were usually able to get the answer correct. Answers were immediately graded by the online program as they submitted their answers, giving students immediate feedback and scores.

The second college algebra course was run by the Mathematics Education Department, specifically overseen by Dawn Teuscher. This course used the *Pathways: College Algebra* textbook and curriculum (Carlson, 2013). The *Pathways* curriculum was developed by a team of mathematics educators, which included Dawn Teuscher, and was overseen by Arizona State University Mathematics Education professor Marilyn Carlson. The team worked together to improve college algebra, and better prepare students for calculus by "focusing on aspects of mathematics that research has found students don't understand and need to understand [in order] to be successful in calculus" (Huchendorf & Smith, 2013, p. 22). These aspects are what Dawn Teuscher calls the five foundations of calculus: function notation, composition of functions, inverse functions, rate of change, and solving functions and equations (Huchendorf & Smith, 2013).

These five concepts for success in calculus were initially emphasized by Carlson, Oehrtman, and Engelke (2010) in their taxonomy of foundational knowledge for beginning calculus. This taxonomy was created through a series of studies aimed at understanding the reasoning abilities and understandings that are fundamental to success in calculus (Carlson, 1995, 1997, 1998; Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Carlson, Larsen, & Lesh, 2003; Engelke, 2007; Oehrtman, 2004, 2008a, 2008b; Oehrtman, Carlson, & Thompson, 2008), and consists of three categories of *Reasoning Abilities* and three categories of *Understandings*. The *Reasoning Abilities* include (1) a process view of function (being able to conceive of a function as an entity that accepts a continuum of input values to produce a continuum of output values), (2) covariational reasoning (the ability to analyze multiple aspects of how the input and output values of a function change together), and (3) computational abilities (facility with manipulations and procedures that are needed when evaluating algebraic representations of functions and solving equations). The *Understandings* include (1) understanding the meaning of function concepts (including function evaluations, rate of change, function composition, and inverse functions), (2) understanding the growth rate of function types (linear, exponential, rational, non-linear), and (3) understanding function representations and being able to interpret, use, construct, and connect them. The importance of these reasoning abilities and understandings was shown by developing a tool for assessing them, the Precalculus Concept Assessment (PCA), which demonstrated that students with these abilities and understandings were more successful in calculus courses (Carlson et al., 2010).

Based upon this taxonomy of foundational knowledge for beginning calculus, Marilyn Carlson wrote *Pathways*, a college algebra curriculum to help students develop these key reasoning abilities and understandings (Carlson, 2013). As stated in the introduction to the text:

> The materials in this workbook are designed with student learning and success in mind and are based on decades of research on student learning. In addition to becoming more confident in your mathematical abilities, the reasoning patterns, problem solving abilities and content knowledge you acquire will make more advanced courses in mathematics, the sciences, engineering, nursing, and business more accessible. The worksheets and homework will help you see a purpose for learning and understanding the ideas of algebra, while also helping you acquire critical knowledge and ways of thinking that you will need for learning mathematics in the future (Carlson, 2013, p. iii).

Dr. Dawn Teuscher further emphasizes that 90% of the *Pathways* curriculum is set within a contextual situation that students can connect with, while focusing on the fundamental concepts students need in order to understand calculus (Huchendorf & Smith, 2013).

Importantly for this study, the design of the *Pathways* curriculum (Carlson, 2013) should be more conducive to helping students develop productive dispositions towards mathematics than a traditional college algebra course. As Carlson (2013) said in the introduction to the *Pathways* text, the curriculum is designed to improve students' mathematical confidence (promoting the Self-Efficacy Component), help students see purpose for learning and understanding mathematics (promoting the Math as Sensible Component), and enhance students problem solving abilities (promoting the Perseverance Component). Furthermore, the *Pathways* course has many of the elements of classrooms that should help students develop a productive disposition described by McClain and Cobb (2001) and Jansen (2012), such as collaborative learning, a focus on conceptual understanding, and a transfer of responsibility of learning to the students. Thus, studying the effects of *Pathways* on the mathematical dispositions of students, as well as the effects of a traditional college algebra course, can be particularly useful for bettering our understanding of specific aspects of college algebra courses that affect students' productive dispositions.

The course, although run by the Mathematics Education Department, still operated under the Mathematics Department; as such, students in the course were still required to take the college algebra final exam that was given by the Mathematics Department. However, the *Pathways* course did not cover all of the same topics as the traditional course did, as the course was much more focused on the topics students need to understand before taking Calculus (Carlson, 1995, 1997, 1998; Carlson et al., 2002; Carlson et al., 2003; Engelke, 2007; Oehrtman, 2004, 2008a, 2008b; Oehrtman et al., 2008). The main topics covered by this college algebra course were:

1. Proportionality—Students learn about quantities, co-variation of quantities, proportionality, constant rate of change, linearity, and average speed.
2. Functions—Students explore tables, formulas, and graphs, and then learn about function notation, function composition, representing function composition, constraint problems, and inverse functions.
3. Exponential and Logarithmic Functions—Students explore the meaning of exponents, and compare linear and exponential behavior. Students then learn about factors and creating formulas, growth and decay, compounding periods and compound interest, and the motivation behind the natural constant e.
4. Polynomials—Students explore polynomial functions and their behaviors, imaginary numbers, the quadratic formula, complex roots, and conjugates.
5. Rational Functions—Students learn about rational functions, vertical asymptotes, the end

behavior of rational functions, graphing and limits (Carlson, 2013).

As the students are required to take the Mathematics Department final, *Pathways* students also briefly work on systems of equations, as well as probability, sequences, and summations. Figure 2 summarizes and compares the topics covered in both the traditional and *Pathways* courses.

Topics in the Traditional College Algebra Course ("Math 110: College Algebra")	***Pathways* College Algebra (Carlson, 2013)**
Functions and their Graphs	Proportionality—Students learn about quantities, co-variation of quantities, proportionality, constant rate of change, linearity, and average speed. Functions—Students explore tables, formulas, and graphs, and then learn about function notation, function composition, representing function composition, constraint problems, and inverse functions.
Polynomial and Rational Functions	Polynomial Functions—Students explore polynomial functions and their behaviors, imaginary numbers, the quadratic formula, complex roots, and conjugates. Rational Functions—Students learn about rational functions, vertical asymptotes, the end behavior of rational functions, graphing and limits
Exponential and Logarithmic Functions	Students explore the meaning of exponents, and compare linear and exponential behavior. Students then learn about factors and creating formulas, growth and decay, compounding periods and compound interest, and the motivation behind the natural constant *e*.
Conic Sections	Taught briefly at the end of the course to satisfy requirements imposed by the Mathematics Department to prepare for the Mathematics Department final exam.
Systems of Equations and Inequalities	
Sequences and Induction	
Counting, Probability, and the Binomial Theorem	

Figure 2. Comparison of topics taught within the two college algebra courses.

By focusing on fewer topics and spending more time on what they did study, the *Pathways* course tried to help students develop conceptual understanding and see meaning behind the mathematics they were learning.

Pathways sections met three times each week with their instructor for one hour, for a total of three hours. The classes had 30-40 students, enabling the class to be much more interactive, with group and whole class discussions being held frequently. Students were strongly encouraged to participate and learn from one another, and there was a heavy focus on helping all in the class develop a strong conceptual understanding. Students were seen as understanding a topic when they could clearly express their own reasoning and were able to justify that reasoning satisfactorily; being able to produce the correct answer by itself was insufficient. Tests and assignments in the course therefore required students to consistently explain and justify their thinking. As long as the reasoning was coherent and mathematically sound, students were given full credit. No particular preferred method or procedure was given; rather, students were given the tools they needed to think about and reason through problems and make sense of the mathematics they were using.

The textbook for the *Pathways* course consisted of two parts. The first part was a physical workbook wherein students worked on explorations of the mathematical concepts, and engaged in problem solving. These explorations were most often done in groups within the classroom. The workbook also contained homework problems students were assigned to work through outside of class to help solidify the mathematical concepts explored and taught in class. The second part of the textbook was online, and consisted of readings and learning modules students were supposed to read and explore before attending the class where the concepts would be further taught. After students did the online reading, they were required to take an online quiz to assess whether or not they had read and understood the online textbook sections required for each particular class. Quizzes and homework were graded on completion, but the instructors would give written feedback on their homework, pointing out mistakes and helping the student correct them, or giving suggestions on how students could make their arguments stronger.

Teachers for both classes mainly consisted of graduate students, with the traditional course taught by graduate students from the Mathematics Department (with the exception of the evening section mentioned previously, which was taught by a mathematics teacher from the Mathematics Department), and the *Pathways* course taught by graduate students from the Mathematics Education Department. Teacher assistants (TAs) for both courses were mainly undergraduate students. In the *Pathways* course, these undergraduates helped with grading, or held office hours where students could come and get specific help on the problems within the *Pathways* curriculum. In the traditional course, the undergraduate TAs were usually in charge of the lab taught twice a week, and also held some office hours for students to get help. Students in both courses also had access to an open mathematics lab at BYU. This is a large room with seating for 50-100 students, where students sit at tables and get help from math lab workers who are there to help with almost any undergraduate mathematics course.

Enrollments in the two classes were fairly comparable, with 225 students enrolled in the *Pathways* sections of college algebra, and 148 in the traditional sections of college algebra. For the purposes of this study, efforts were made to get a representative sample from both courses so that comparisons could be made between them. However, as will be shown within the next section, difficulties arose in attempting to work with some of the sections of the traditional course.

Participants and Data Collection

The study I conducted was a mixed methods study, where I used four data collection methods: a five-point, “Likert-type scale” survey measuring students’ dispositions; E-mail interviews with purposefully selected students; an open ended question survey given to all students in both classes at the end of the course; and follow-up surveys with students who had large changes in their dispositions or interesting responses on the surveys. In the next few sections, I will expound upon and describe the purpose of these different methods.

Mathematical disposition survey. Using research that has been conducted by others, I gathered a list of thirty-eight statements aimed at measuring the Self-Efficacy, Math as Sensible, and Perseverance components of a mathematical disposition. These statements were mainly taken or adapted from surveys created by other researchers. For the Self-Efficacy component, statements were gathered from the Fennema-Sherman Confidence in Learning Mathematics Scale (Fennema & Sherman, 1976), studies by Usher and Pajares (2009) and Berger and Karabenick (2011), and Diana K. May's Dissertation at the University of Georgia (May, 2009) and include statements like "I am sure that I can learn mathematics." For the Math as Sensible component, statements were collected from the Fennema-Sherman Usefulness of Mathematics Scale (Fennema & Sherman, 1976), and the Indiana Mathematics Belief Scales (Kloosterman & Stage, 1992) and include statements like "I will need mathematics for my future work," and "Time used to investigate why a solution to a math problem works is time well spent." For the Perseverance component, statements were pulled directly from the Indiana Mathematics Belief Scales (Kloosterman & Stage, 1992), and include statements like "I find I can do hard math problems if I just hang in there," and "I can get smarter in math by trying hard." Questions were pulled directly from the above resources, giving me mixed parity of the statements, with about one-fourth of the statements worded negatively and the rest worded positively. All of the survey statements, including the parity of each statement, can be found in Appendix A.

In order to establish internal validity of this survey, I conducted a pilot study, giving an initial version of the survey (which can be found in Appendix B) to 47 students in my Winter 2014 first semester Calculus labs at Brigham Young University. Cronbach's Alphas were calculated for each of the Productive Mathematical Disposition Scale's components. The results are shown in Table 1:

Table 1

Cronbach's Alphas for Mathematical Disposition Survey Scales from Pilot Study

Component	Number of Items	Cronbach's Alpha
Self-Efficacy Component	12	.904
Math as Sensible Component	14	.872
Perseverance Component	12	.888

With these relatively high Alpha scores, I felt fairly confident that my subscales were internally consistent (Tavakol & Dennick, 2011). Thus, after the pilot study, the only change made was the wording of statement 22, "Working can improve one's ability in mathematics," suggested by a student in one of the Calculus labs. The final version of this survey has the following wording for statement 22: "Working hard in a math class can improve one's ability in mathematics." This final version of the Mathematical Disposition Survey can be found in Appendix C. Later, the Chronbach's Alphas were calculated for the three scales (Self-Efficacy, Math as Sensible, and Perseverance) on this final version of the survey, using the results of the actual study, and are shown in Table 2 below.

Table 2

Cronbach's Alphas for the Mathematical Disposition Survey

Scale	Chronbach's Alpha
Self-Efficacy	0.914
Math as Sensible	0.914
Perseverance	0.895

These relatively high alpha scores increased my confidence that the scales I had developed to measure students' self-efficacy beliefs, beliefs about the sensibleness of mathematics, and beliefs about the importance of hard work and perseverance, were internally consistent (Tavakol &

Dennick, 2011) as they were even higher than those in the pilot study I had conducted.

Students from both traditional college algebra courses and the *Pathways* courses were surveyed at the beginning and end of the semester. In order to maximize the number of students taking the pre-course survey, the survey was first given in class (to as many students as possible), and secondly online (for those who missed the in class survey, or for classes that I was not able to give the survey in class). The survey was voluntary, but with two opportunities to take the survey, I hoped to be able to collect data from as many students as possible in both classes. The post-course survey was done solely through *Qualtrics*, an online survey program, and was sent out through E-mail to those who took the pre-course survey. There were 48 traditional students who took the pre-course survey, and 191 from the *Pathways* course. In the end, I only analyzed data gathered from students who completed both the pre-course and post-course surveys, since I was interested in analyzing how the students' dispositions changed after completing the course. This gave me 18 students within the traditional courses, and 56 from the Pathways courses.

Recruiting participants. Participants for this study were initially recruited by my contacting instructors of the college algebra courses individually, and requesting to go into their class at the very beginning of the semester to elicit the help of students on the study. For the traditional course, I personally knew one of the instructors, Joseph (*pseudonym*), and he was very enthusiastic about having me come. Additionally, the instructor of the evening section of college algebra was also more than willing to have me come into her class to elicit participants. However, the third traditional college algebra instructor did not want me to come to their classroom at all. I asked this third instructor to forward the electronic version of the survey on to his students, but only a handful from his class ended up participating. Additional resistance from the traditional classes arose from the professor in charge of Math 110: College Algebra. When he found out that I had gone into the college algebra classes to elicit participants for my study, he actually told Joseph not to let me come back into the class ever again. In hindsight, it would have been a good idea to inform the mathematics professor in charge of college algebra about my study first, before talking with the instructors. This might have opened more of an opportunity to get traditional students to participate. However, because of the resistance from the Mathematics Department, it greatly limited the sample size of students in the traditional course.

In contrast, all three *Pathways* instructors were fellow graduate students with myself, so attending these classes was fairly simple. Furthermore, the *Pathways* instructors were much more enthusiastic about my study, and may have encouraged their students to participate. In addition to this, Dawn Teuscher, the Mathematics Education professor who oversees *Pathways* at BYU, was very interested in my study, and might have had an influence on encouraging students to participate, either explicitly or implicitly.

In addition to resistance or cooperation from the instructors and heads of the two college algebra courses influencing the number of participants, another influence was whether the survey was given by me in person, or through an E-mail invitation to participate. Students were much more likely to take the survey when I was asking them to participate in person within the classroom, rather than through an E-mailed version of the survey. Thus, because I only gave the post-course survey through E-mail to students who had participated in the initial survey, this may have contributed to the large drop in the number of participants from the pre-course to post-course survey.

The purpose of the survey was to see if there was significant change in students' mathematical dispositions as they went through the college algebra course. Further, I was also able to compare the students within the two courses, and see if there was a significant difference between the effects of the traditional college algebra course and the *Pathways* course on students' dispositions.

E-mail interviews. After the initial survey was given, willing students were selected from the traditional course (8 students) and the *Pathways* course (10 students) to participate in interviews, constituting 18 students in total. I attempted to select the students from each class purposefully based on the Mathematical Disposition Survey, so that I would be able to interview students with different levels of mathematical dispositions (low, medium, and high). In order to find the low, medium, and high students, an overall sum score of their disposition was found for each student by first correcting their responses for parity, and then adding up all 38 questions on the pre-course Mathematical Disposition Survey. These scores were then ordered, and I chose students whose scores fell into the lower third of the class, middle third of the class, and upper third of the class. This selection process, however, was limited, due to the fact that only a few of those who I personally selected responded back to my initial E-mails, and thus I was not able to get as many students as I had hoped for. More specifically, I only had about one half of the amount I had intended. Thus, I ended up sending an E-mail out to all students who had indicated that they would be willing to participate in these interviews, and this gave me the rest of the participants for this portion of my study. Fortunately, I was still able to get students in the low, medium, and high mathematical disposition range from both classes.

Besides the limitation that resulted from difficulty of selecting students, another limitation was the fact that not all students responded to each E-mail interview that I sent out. All 18 students responded to the first E-mail interview, but the response rate dropped throughout the semester with each successive interview. In particular, for the last three interviews, I had 10-12 students responding. As some explanation to this, during the study, two students (Becky and

Dina) ended up withdrawing from the course (one in the *Pathways* course and one in the traditional course); however, there were a few students who only responded to a few of the interviews. A list of students (*all names are pseudonyms*), their course, initial mathematical disposition level, and how many interviews they responded to, are included in Figure 3.

Participant	**College Algebra Course**	**Major Declared at the Beginning of the Course**	**Initial Mathematical Disposition Level**	**Number of Interviews responded to**
Joe	Pathways	Communications	Low	9
Rick	Traditional	Undecided	Low	7
Jon	Pathways	Psychology	Low	9
Finn	Pathways	Physiology	Low	9
Abe	Traditional	Business Management	Low	8
Todd	Traditional	Plant and Wildlife Sciences	Low	8
Mark	Traditional	Pre-Animation	Low	6
Becky	Pathways	Biology	Medium	5
Teri	Traditional	Special Education	Medium	6
Kylie	Pathways	Psychology	Medium	8
Luna	Pathways	Pre-Management	Medium	8
Jeff	Pathways	Economics	Medium	2
Coco	Pathways	Environmental Science	Medium	8
Ferven	Traditional	Business Management	High	8
Dina	Traditional	Pre-Management	High	4
River	Pathways	Animation	High	9
Ruth	Pathways	Elementary Education	High	3
James	Traditional	Computer Engineering	High	8

Figure 3. List of participants in the E-mail interviews.

There were nine interviews sent out during the course of the semester. Each interview consisted of four to five open ended questions, which had two specific purposes: first, to attempt to observe and examine how a students' mathematical disposition changes as a student moves through a college algebra course; and second, to find out what specific aspects of the course may have been an explanation for this change in their mathematical dispositions. A list of the E-mail interviews is included in Appendix D. The students who participated in at least 7 of the 9 interviews were given a $10 BYU gift card as compensation for their time.

Open-ended survey—College Algebra Experiences survey. Based on the responses that I had received to specific questions during the E-mail interviews, I purposefully selected nine questions from the interviews to send out to all of the students who had taken the initial pre-course Mathematical Disposition survey and had given me their E-mail. This open-ended survey was meant to see if the patterns (if there were any) among the interviewed students were also common among the whole class. Participation in this survey was completely voluntary. Eighteen students in the *Pathways* course, and four in the traditional course chose to participate in this survey. This survey was done completely online through *Qualtrics*, and screenshots of the survey are included in Appendix E.

Follow-up survey. Students who had significant changes in their mathematical dispositions, or students who had interesting and intriguing responses on the E-mail interviews or the College Algebra Experiences survey, were E-mailed to see if they would be willing to participate in a follow-up survey after the course was completely finished and grades were posted. These surveys were completely voluntary, and were individualized for each student, as an attempt to find out more in depth details about the students' mathematical disposition and their experiences with the college algebra course.

By this point in my study, however, students were actually getting frustrated that I was still contacting them. I even had one student respond back to the invitation with great hostility, telling me that they had already taken the surveys, and that I was angering them by trying to contact them so much. As such, I only had two students participate in the follow-up surveys. The first student, Abe, was a participant in the E-mail interviews from the traditional course, and mentioned that he had actually taken the *Pathways* course previously, but enjoyed the traditional course much more. In asking Abe questions about both of his college algebra experiences, I hoped to be able to discover why he did not like the *Pathways* course, but enjoyed and succeeded in the traditional course. The other participant, Nadine, studying Psychology as her major, participated in the *Pathways* course and was selected because she had a large increase in her mathematical disposition, and I wanted to study why this was the case.

In order to be less of a burden on students and more accommodating to their schedules, as well as make it more likely for students to participate (although this did not seem to help), these surveys were conducted through E-mail once again, and a list of the questions asked to the two students is included in Appendix F.

Data Analysis

The analysis of the Mathematical Disposition Survey focused on two aspects: the change in the mathematical disposition of individual students in the classes, and how the dispositions of students differ between the two classes. In order to accomplish this, the data of all students who participated in the pre- and post-surveys was compiled into Excel, and the gain score for each question was calculated for each student by subtracting the pre-course answer (a number 1-5) from the post-course answer (a number 1-5), correcting for parity as needed. Next, a sum of the gain scores for all questions that dealt with a specific component of mathematical dispositions was calculated for each student, so that I had an overall sum of the gain scores on each component for each student. These gain scores were subjected to a multivariate analysis of variance (MANOVA), using the different college algebra classes as the independent variable, and the gain score sums of the three different scales (Self-Efficacy, Math as Sensible, and

Perseverance) for each student as the dependent variables. This analysis allowed me to see if the college algebra courses had an effect upon individual students' mathematical dispositions. It further gave me the ability to compare differences between the two classes, to see if there was a significant difference in their effects on students' dispositions.

In addition to the MANOVA of the Mathematical Disposition Survey, one sample t-tests were conducted with comparison to a mean gain score of zero (indicating no change) on the gain scores of all individual questions, on the gain score sums of the three components of a mathematical disposition (Self-Efficacy, Math as Sensible, and Perseverance), and on an overall disposition gain score found by creating a sum of the gain scores for all 38 questions. This was done to see if the changes in students' dispositions within the two courses could be considered statistically significant from zero (i.e. statistically significant from no change in their disposition). Furthermore, a Brown Forsythe Robust test of equality of means was conducted on the mean gain scores of each individual question to see if there were statistically significant difference between the two courses on individual questions. Lastly, because of some surprising results in the Math as Sensible component, I attempted to break up the questions related to this component into three sub-components of usefulness, sensibleness, and worthwhileness, and one sample t-tests were conducted with comparison to a mean gain score of zero on each of these sub-components. The questions were separated as shown in Table 3.

The analysis of the interviews, the open-ended survey, and the follow-up surveys, were aimed at identifying specific aspects of the college algebra courses that might have had an effect upon students' mathematical dispositions, and at gaining insight into any differences discovered through the questionnaire. Because the interviews and surveys were done through E-mail or online programs (*Qualtrics*) I had immediate access to students' responses in textual form. I first focused on an initial coding of the E-mail interviews. Each students' responses were coded with one of four codes, three based on the three components of a mathematical disposition, and one focused on the particular aspects of the students' college algebra experience:

MS= Response dealing with the Math as Sensible component.

SE= Response dealing with the Self-Efficacy component.

P= Response dealing with the Perseverance component.

AC= Response that points out particular **aspects of the course** that might be affecting their dispositions (likes or dislikes of the course, specific things mentioned in correspondence with the components of mathematical dispositions, etc.).

All students' responses were then organized into the four code categories, separated by the two different college algebra courses (*Pathways* or traditional). I then looked for commonalities and patterns among the students within each class, focusing on specific aspects of the course that were frequently mentioned by students, and that may have had an effect on the students' dispositions. These possible effects were identified by looking for instances where students mentioned a particular component of their mathematical disposition changing or being influenced by certain aspects of the course.

After initial sorting, a second round of coding took place on the responses initially coded for **aspects of the course**. This round focused on commonalities and differences in the first codes, and from it emerged more fine-grained coding of commonalities among the aspects that were commonly mentioned by students as being significant. Codes used for this level of analysis were:

CS = Classroom Structure – Responses that mentioned particular ways the classroom was organized, and classes were conducted, that had either positive or negative effect on their college algebra experience.

T = Teacher – Responses that mentioned the teacher having a positive or negative effect on their experience.

TB = Textbook – Responses that mentioned the particular textbook the students were using.

H = Homework – Responses that mentioned their like or dislike of the homework

TM = Teaching Method – Responses that showed students looked for a particular teaching method to help them understand and succeed in the class.

RWA = Real-World Application – Responses that mentioned the helpfulness, or hindrance caused by, the concepts being shown within real-world applications.

Tests = Responses that mentioned the fairness or difficulty of the tests.

UC = Useful concepts – Concepts students mentioned as being personally useful or not useful.

After this finer level of coding, commonalities among students, and contrasts between the *Pathways* and traditional college algebra courses, were identified and recorded.

I next examined the responses to the open-ended survey and the follow-up interviews as a check for validity of the commonalities I had found within the interviews. By seeing if other students in the classes had similar ideas to those I had interviewed, I could check to see if the patterns I had identified were also common among the rest of the class. Responses to the open-ended survey were analyzed using the coding scheme developed from the E-mail interviews, as described above. The follow-up interviews (of which there were only two) were viewed as miniature case studies of the college algebra experience of two particular students, one each within the *Pathways* and traditional courses.

As a last part of the analysis, comparisons between the quantitative results from the Mathematical Disposition Survey and the qualitative results of the E-mail interviews, open-ended survey, and follow-up survey were made. In doing so, I looked for possible explanations for the results found in the Mathematical Disposition Survey within the students' responses, focusing on reasons why it might make sense that students' dispositions rose or dropped within particular questions, within a particular component of a mathematical disposition, or on their overall mathematical disposition scores. This gave me the ability to make conclusions about ways that dispositions can develop and change, as well as advocate for further action and research needed in the future on this topic. Further, it gave me the opportunity to show how my research can help college algebra teachers improve students' mathematical dispositions.

Chapter 4: Results

In this chapter, I present the results of my research. First, I present results related to the effects of college algebra on: (1) students' beliefs about the degree to which mathematics is sensible, useful, and worthwhile; (2) students' beliefs about the importance of hard work and perseverance to mathematical success; and (3) students' mathematical self-efficacy beliefs, or their beliefs about how successful they will be in using mathematics in the future. Next, additional insights about college algebra and its overall effect on students' mathematical dispositions will be shared. Lastly, in the discussion section, I will explain how the results of this study can be connected with and situated within previous research on students' mathematical dispositions, as well as additional insights this study has given the mathematics education community about how dispositions grow and develop, specifically within college algebra.

For ease of reading, when quotes are used from students' responses to the interviews or follow-up surveys, their name will be given followed by a *P* in italics for those who were in the *Pathways* course, and a *T* in italics for those in the traditional course. On the open-ended survey, students were simply given numbers in the order of which they responded to the online survey, as the survey was completely anonymous; thus, when using quotes from the open-ended survey, *PS#* will indicate a pathways student's response on the open-ended survey, and *TS#* will indicate a traditional student's response.

The Effects of College Algebra on the Math as Sensible Component

The descriptive statistics for the gain scores of the Math as Sensible component within both classes are presented in Table 3.

Table 3

Results from the MANOVA of the Mathematical Disposition Survey -- Math as Sensible Component

Scale	Class	Mean Gain Score	Standard Deviation	Significance Level of Difference between Courses
Math as Sensible	*Pathways*	-1.93	6.051	0.798
	Traditional	-2.33	5.018	

From the MANOVA results, it seems that, on average, students' beliefs about the degree to which mathematics is sensible, useful, and worthwhile dropped within both classes. Furthermore, there was not a statistically significant difference between the two courses in their effects on this component of students' mathematical dispositions. However, this only tells a part of the story about the ways in which the two courses effected students' beliefs about the sensibleness of mathematics. In fact, when conducting one sample t-tests with the Math as Sensible component for the two courses, only the *Pathways* course mean gain score was found to be statistically significant from zero, $t(55) = -2.385,\ p = .021$, 95% CI [-3.55, -0.31], suggesting that the *Pathways* course had a negative effect on the Math as Sensible component of students' mathematical dispositions.

To further explore reasons why *Pathways* students' beliefs about mathematics being sensible trended negatively, I looked for individual questions dealing with the Math as Sensible component of mathematical dispositions which had mean gain scores that were statistically significant from zero. Two questions showed this statistically significant difference, summarized in Table 4.

Table 4

Math as Sensible Questions which had Mean Gain Scores that were Significantly Different from Zero in the Pathways Course

Question	t	df	Significance	95% Confidence Interval	
				Lower	Upper
#7 Understanding why the answer works is more important than getting a right answer in math (Parity Corrected)	-4.371	55	<0.001	-0.742	-0.276
#38 A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem	-2.726	55	0.009	-0.62	-0.09

This led to an interesting observation. Although one of the main goals of the *Pathways* course is to get students to reason about mathematical concepts for themselves, and then improve the students' abilities to explain and justify their solutions, it seemed that students, on average, felt that a correct answer in mathematics should be good enough. In fact, when looking at some of the results of the interviews and surveys, this became even more apparent. There were a few students in the *Pathways* course, when asked what it means to understand a mathematical concept, who were solely focused on getting answers:

PS 12 … [to] understand means you can do the math problem. Period.

Jon (P) [To understand means] that I can get the correct answer.

…

I don't care whatsoever about the meaning of the things I'm doing in math, especially when I can't solve it. Teach me the mechanics first and meaning will come with time.

Other *Pathways* students felt that the grading procedures that so heavily focused on justification and explanations, rather than correct answers, were unfair and ill-explained:

River (P) [I wish we had more] resources and examples to meet the exact expectations of *explanations* and *work* on tests. (Emphasis added).

...

The exactness of procedures with little to no room for innovation or *more efficient ways to achieve results after a complete knowledge is displayed of a concept* can be frustrating. (Emphasis added).

PS 18 [Grading was] very subjective. Even if we had the right answer, they would sometimes take off points for the way we solved it.

Arguably, many of the students who come into the *Pathways* course were probably coming from K-12 mathematics courses that were more traditional in nature, where grading and success are mainly products of whether or not students are able to provide correct answers. When these students come into the *Pathways* course, and find they are being penalized for poor justification and explanations, even when they have the correct answer, they are frustrated and annoyed. Thus, the negative gain scores on these two questions could have been part of the reason why students' dispositions in the Math as Sensible component dropped within the *Pathways* course.

As further explanation of the drop in the Math as Sensible component among *Pathways* students, I also looked at the separation of usefulness, sensibleness, and worthwhileness into three subscales. In running one sample t-tests on the gain score sums within these subscales, only the gain score mean on the usefulness subscale showed a statistically significant difference from zero, $t(55) = -3.258$, $p = 0.002$, 95% CI [-2.379, -0.567], suggesting that students in the *Pathways* course generally left the course feeling that mathematics was less useful than when they began the course. In looking at the interviews and surveys, a commonality among students' responses that could be an explanation for this result was the fact that some concepts taught within the college algebra course were seen by students as useless or redundant:

Finn (P) Rational functions I don't feel has any real life application for me, so therefore all of math is not relevant to me. Imaginary numbers showed me that math doesn't always follow the rules such as you can't divide by 0, in this example it shows how you can break the rule and square root a negative; that's just how I see it anyway.

Joe (P) I don't understand how some of the material will be applicable in my life.

Jon (P) Take [this class] away from the "required" section of academia and replace it with a class that actually teaches you something valuable. Like how to take out a car loan, etc. Teach us how to use the software that solves problems and let us on our way.

PS 6 Too much time [was] spent on things that should have been prior knowledge and hardly any time spent on real concepts.

In addition, many of the *Pathways* students were frustrated by the amount of material that was only briefly covered at the end of the course, in preparation for the Mathematics Department Final exam:

PS 1 I thought the math department [final] was terrible. We were not prepared to take it at all, and I think you could tell that from a lot of the scores.

Luna (P) [One of my least favorite aspects of the course is] the final [Mathematics Department] test we have been preparing for … I just don't feel like we've had enough time to go over all of the stuff we are supposed to know.

Thus, although *Pathways* tries to help students see mathematics as relevant and useful, particularly by contextualizing 90% of the material and mathematical concepts taught, it seemed that students still struggled to see how certain concepts of mathematics could be personally useful. In particular, the concepts briefly taught at the end of the course, in preparation for the Mathematics Department final, might have been some of the concepts students struggled to see the usefulness of, since the only purpose in covering those concepts was for the final exam.

One last possible reason the Math as Sensible component of *Pathways* students' mathematical dispositions dropped, particularly their views of the usefulness of mathematics, are their views of the *Pathways* textbook. A great number of *Pathways* students mentioned that the online textbook was one of their least favorite elements of the course, and many even felt that the online textbook was completely useless:

Coco (P) I don't like the textbook, the readings do not seem to be very helpful, and the layout is not very user friendly.

Joe (P) I didn't like that the textbook was online. That automatically made me have ZERO desire to look up the problems and answers for myself. I would have loved to easily flip back a few pages in a chapter and review a concept than have to go online and attempt to navigate the website.

PS 13 I absolutely loathed the textbook/workbook. I found it to be useless as well as the furthest thing from clear.

As students saw the textbook as useless within the *Pathways* course, this could have easily affected students' beliefs about the usefulness of mathematics in general.

These results from the *Pathways* students are surprising considering that one of the most frequently mentioned aspects of the *Pathways* course that students enjoyed was the abundance of real-world applications of the concepts being taught within the course:

Joe (P) I appreciate the various styles of problems that we do in order to help us better understand. For example, with our recent section, we studied exponential functions. In those problems, we talked a lot about applying those functions to real life situations, like APR, population, etc. Things that help us to better understand WHY you would ever need this knowledge again.

River (P) The teaching format focused on application is good, and helps me not just know the concepts, but actually learn them and learn to use them.

Finn (P) The content having real-life application and focusing on those [is one of my favorite aspects of the course].

Coco (P) It's … cool to see how [mathematics] applies in other courses and areas of life.

Despite students' enjoyment of seeing the mathematical concepts within real-world contexts, the results from the one sample t-tests showed that the *Pathways* class was more definitively harmful to the Math as Sensible component of dispositions than the traditional class, even though the mean gain score on this component was lower within the traditional course.

This is not to say the traditional college algebra course had no effect on the Math as Sensible component of students' dispositions. Perhaps the variance among students was too large to definitively say that the traditional college algebra was overall harmful to the Math as Sensible component of students' dispositions. However, there were two questions on the Mathematical Disposition Survey dealing with the Math as Sensible component that had mean gain scores which were statistically significant from zero. These results are displayed in Table 5.

Table 5

Math as Sensible Questions which had Mean Gain Scores that were Significantly Different from Zero in the Traditional Course

Question	t	df	Significance	95% Confidence Interval	
				Lower	Upper
#24 I will need mathematics for my future work	-3.063	17	0.007	-0.75	-0.14
#28 Understanding the solution to a mathematics problem is more important than obtaining the correct answer. (Parity Corrected)	-2.38	17	0.029	-0.63	-0.04

Thus, for some items focused on the Math as Sensible component, the traditional class was harmful, affecting students' beliefs about the usefulness of mathematics for their future work, and the importance of understanding over correct answers when solving mathematical problems.

Evidence from the interviews and surveys may suggest two reasons for students from the traditional course specifically not being able to see the usefulness of mathematics for their future work. First, students mentioned a lack of real-world applications in the traditional course:

Mark (T) It just doesn't feel very applicable in my life. It gets boring.

James (T) [One of] my least favorite aspects [is] … a lack of more real-world application questions.

Thus, a lack of seeing how the mathematical concepts were useful within the real world may have been some reason why students struggled to see the usefulness of mathematics for their future work. Second, some traditional students mentioned difficulty in seeing the usefulness of some of the content taught within the course, specifically mentioning concepts such as summations as useless.

As for why students' views about the importance of understanding in solving mathematical problems trended negatively, there were not any definitive results from the interviews or surveys that suggest why this may have been the case. However, one speculation might be the nature of the homework within the traditional class. As students are focused solely on getting correct answers to enter into the online homework for credit, they could easily begin to view understanding as being less important to getting correct answers. Furthermore, with the abundant help available built into the online homework, students might have felt at times that it was not necessary to understand.

One last result to present from the interviews and surveys may also help to explain negative mean gain scores on the Math as Sensible component of students' dispositions. When asking students whether or not mathematics was useful to them, the largest influence on their views seemed to be how much they felt their chosen studies or career path could benefit from studying mathematics. Some examples of students' responses will illustrate this result:

Jon (P) I will never be using the mathematics I have learned in this class. I don't know a single person who has. I haven't had to use math up to this point, I don't know why anything would change. I have observed those that work in the jobs I want, and none of them use math either. It's a useless practice for me.

PS 10 I personally don't care for it, and I most likely won't be applying higher division math in the future.

Mark (T) [KW: Do you think you will use the mathematics learned in this course?] To be honest? Not really. I'm an art major.

Ferven (T) I do think I will use what I learn in [this] class when I am analyzing data in my job.

Joe (P) I feel as though the things we've learned so far will be useful. Understanding functions and their different varieties will be helpful if I am to pursue a career in business.

River (P) As an animator, [mathematics is] useful in the construction of models. As an author, they help me better understand concepts outside of the realm of my expertise.

Hence, it may be that students' beliefs about the usefulness of mathematics were more closely tied to their chosen field of study, rather than the particular college algebra course they were in. Furthermore, since more and more students are taking college algebra as their last mathematics course (Small, 2006), it can be reasonably argued that a vast majority of students taking college algebra are probably not going into mathematically intensive careers. In this case, college algebra might actually reaffirm to students that mathematics is not useful to them as they study mathematical concepts which they cannot see as useful to their chosen career path.

The Effects of College Algebra on the Perseverance Component

As we move to the Perseverance component of students' mathematical dispositions, a different story begins to unfold. The descriptive statistics for the gain scores of the Perseverance component within both classes are presented in Table 6.

Table 6

Results from the MANOVA of the Mathematical Disposition Survey -- Perseverance Component

Scale	Class	Mean Gain Score	Standard Deviation	Significance Level of Difference between Courses
Perseverance	*Pathways*	-0.86	5.603	0.085
	Traditional	-3.50	5.447	

With these results from the MANOVA of the Mathematical Disposition Survey, it looked as if both courses had, on average, a negative impact on the Perseverance component, with no statistically significant difference between the two courses. However, upon further examination of the data through the use of a one sample t-test of the mean gain score of each class in comparison to a mean of zero, only the traditional college algebra course was statistically significant from zero, $t(17) = -2.699,\ p = 0.015$, 95% CI [-6.24, -0.76], suggesting that the traditional college algebra course had an overall negative effect on the Perseverance component of students' mathematical dispositions.

To gain further insight into the reasons why the traditional course had a negative effect on the Perseverance component, I looked at the individual questions from the Mathematical Disposition Survey specifically dealing with the Perseverance component which had mean gain scores that were statistically significant from zero. Two questions were found, and the results of their one sample t-tests are given in Table 7.

Table 7

Perseverance Questions which had Mean Gain Scores that were Significantly Different from Zero in the Traditional Course

Question	t	df	Significance	95% Confidence Interval	
				Lower	Upper
#3 I'm very good at solving math problems that take a while to figure out. (Parity Corrected)	-2.715	17	0.015	-0.69	-0.09
#6 I can get smarter in math by trying hard	-3.117	17	0.006	-1.12	-0.22

This suggests two possible reasons the Perseverance component of traditional students' mathematical dispositions may have, on average, gone down. First, it seems like students were given evidence within the traditional course that mathematical problem solving which takes a long time is much more difficult, even sometimes impossible. This evidence may have come from the online homework students worked on within the traditional course. Arguably, this online homework could be considered to consist of what Schoenfeld (1989) calls bite-sized exercises aimed at achieving mastery. The problems were meant to be short applications of the concepts taught within class, which should only take a few minutes to complete. Furthermore, if students struggled to solve the problem within a few minutes, they could quickly click the "Hint" or "Help me solve this" button built into the online homework, giving the student access to step-by-step walkthroughs of how to procedurally solve the problem. Evidence of students' use of these tools, and reliance upon them, were found within the interviews:

Teresa (T) If I have trouble with a problem, I usually use the "Help Me Solve This" tool on the website …

Abe (T) I like the way homework works in class. … I love that I can get tips on how to solve the questions …

TS 2 I … liked that the homework problems had step-by-step instructions on how to do each problem.

With the online homework problems designed to be short exercises, and the ease of getting help to solve these problems, it can be easily argued that the traditional students rarely had the opportunity to engage in productive struggle (NCTM, 2014), and as a result, rarely saw success in persevering through a difficult mathematics problems and seeing success at the end of the struggle. Additionally, the most likely place students encountered problems that required time, thought, and a bit of struggle were on the exams within the traditional course, which many students mentioned being overly difficult and unfair. Because students cannot get any help or assistance during an exam, struggling through difficult problems and seeing little to no success in a high-stress situation could easily have been harmful to students' beliefs about their abilities to work through mathematical problems which take a long time to figure out. More details about the difficulty of the traditional tests will be given later in the Self-Efficacy results section.

The second possible reason traditional students' beliefs about the importance of hard work and perseverance were negatively affected are the experiences students had, or observed other students having, with regard to the amount of effort they put into the class, and the levels of success they achieved. These experiences might have led students to believe, or reinforced their beliefs about the advantages that arise from having innate ability in mathematics:

TS 2 Hard work and perseverance will take you very far. However, being a "math person" gives you a huge benefit and head start in the subject.

As students witnessed those with seemingly innate mathematical ability performing much better than those who had to work hard, and as they saw how their own efforts within the college algebra course were rewarded, these might have influenced their beliefs about the importance of hard work and perseverance, particularly effort and diligence leading to a person becoming smarter in mathematics.

In comparison, individual questions whose mean gain scores were statistically significant from zero within the *Pathways* course are given in Table 8.

Table 8

Perseverance Questions which had Mean Gain Scores that were Significantly Different from Zero in the Pathways Course

Question	t	df	Significance	95% Confidence Interval	
				Lower	Upper
#3 I'm very good at solving math problems that take a while to figure out. (Parity Corrected)	2.588	55	0.012	0.06	0.48
#6 I can get smarter in math by trying hard	-2.03	55	0.047	-0.39	0
#20 Ability in math increases when one studies hard	-2.03	55	0.047	-0.39	0
#37 If I can't solve a math problem quickly, I keep trying until I see success (Parity Corrected)	-2.192	55	0.033	-0.41	-0.02

The first interesting observation is the positive effect *Pathways* had on students' beliefs about their abilities to solve mathematics problems that take a long time to figure out. Arguably, this may be a result of the *Pathways* course giving students opportunities to engage in productive struggle (NCTM, 2014) within the classroom, which a number of students mentioned as being helpful to their success in the class:

Joe (P) I enjoy the group work; that way, instead of just being lectured on how to solve the problems, we get to work through the problems ourselves and work with each other to understand the material.

PS 11 I … really liked that instead of the teacher lecturing constantly, he would explain what we should do, and then let the students work out the problems on their own with their tables.

By giving students opportunities to see success in problems that take a while to figure out, this small aspect of the Perseverance component increased among the *Pathways* students. However, in odd contrast to the positive effect *Pathways* had on question 3, results showed an opposite, negative effect on question 37, possibly suggesting that *Pathways* students were more likely to quit trying when they could not solve a mathematics problem quickly. Unfortunately, no explanation among the interviews or open-ended surveys could be found for this seemingly contradictory result.

The second observation about these statistically significant Perseverance questions in the *Pathways* course is a similarity with the traditional course in a negative effect on students' beliefs about the feasibility of increasing ability within mathematics through hard work. And, also similar to the traditional course, evidence from the interviews and surveys suggest that these beliefs were largely influenced by the experiences students had, or observed others having, in regards to hard work and effort leading to success. This was made most clear in the follow-up survey with Nadine:

Nadine (P) Hard work is crucial. If you practice and practice, and put in the effort, you are more likely to understand a difficult concept. However, a little bit of talent in being able to think and comprehend the world in a maths-based manner is helpful. I helped several people study and prepare for the exams (people that weren't "math" people); watching them, they put in countless hours on practice and attempts at comprehension; however, their grades never truly improved because they simply couldn't comprehend the way the maths worked. So I would have to say that success is a combination of both [hard work and talent].

This suggests that convincing students that mathematical ability and aptitude can be increased in mathematics through hard work and effort might be more complex than simply engaging students in productive struggle (NCTM, 2014). Although there were these negative effects on individual questions related to the Perseverance component within the *Pathways* course, it is important to remember that the *Pathways* course did not have a statistically significant effect on the overall Perseverance component, whereas the traditional course had a negative effect on it.

The Effects of College Algebra on the Self-Efficacy Component

The starkest contrast between the two college algebra courses involved their effects on students' self-efficacy beliefs. The descriptive statistics for the gain scores of the Self-Efficacy component within both classes are presented in Table 9.

Table 9

Results from the MANOVA of the Mathematical Disposition Survey – Self-Efficacy Component

Scale	Class	Mean Gain Score	Standard Deviation	Significance Level of Difference between Courses
Perseverance	*Pathways*	-0.04	5.985	0.032
	Traditional	-3.50	5.294	

Thus, from the MANOVA of the Mathematical Disposition Survey, the mean gain score of the Self-Efficacy component was the one place where there was a statistically significant difference $(\alpha < 0.05)$ *between* the two courses. In examining the Self-Efficacy component further by looking at one sample t-tests of the mean gain score in comparison with zero, *Pathways* essentially showed no effect on the Self-Efficacy component of students' mathematical dispositions, whereas the traditional class mean gain score was statistically significant from zero, $t(17) = -2.805,\ p = 0.012$, 95% CI [-6.13, -0.87], suggesting that the traditional class harmed students' self-efficacy beliefs.

To further explore why the traditional course was harmful to the Self-Efficacy component, I examined the questions which had gain scores that were statistically significant from zero in a one sample t-test. These are shown in Table 10.

Table 10

Self-Efficacy Questions which had Mean Gain Scores that were Significantly Different from Zero in the Traditional Course

Question	t	df	Significance	95% Confidence Interval	
				Lower	Upper
#25 I am sure that I can learn mathematics	-4.267	17	0.001	-0.91	-0.31
#27 I believe I am the type of person who can do well in mathematics	-2.38	17	0.029	-0.63	-0.04
#35 I can imagine myself working through challenging math problems successfully	-3	17	0.008	-0.85	-0.15

Hence, the traditional course had a negative effect on students' Self-Efficacy beliefs on all three of these statistically significant questions. To examine why this may have been the case, I again turned to the interviews and surveys, and found that the most plausible explanation for the drop in the Self-Efficacy component were the exams within the traditional course. Almost all of the traditional students felt that the exams within the course were overly difficult, not aligned with expectations seen in the homework, and a poor assessment of what they had learned within the class. Examples of this can be seen in the following student quotes:

James (T) [One of] my least favorite aspects would be the difficulty of the tests.

Abe (T) [One of] my … least favorite aspects of this course [is] the lack of continuity between homework/quiz work and the actual tests I'm given.

Ferven (T) I fell like many of the types of questions we learn in class aren't very similar to those on the tests.

Mark (T) I'd change how hard the tests were. Seriously, some of those tests were real doozies.

To explain why these difficult tests may have negatively impacted students' self-efficacy beliefs, I briefly return to my literature review. Two large influences on a students' self-efficacy are students' interpretation of their actual performances (such as how well they feel they did on a test, especially in comparison to others) (Schunk & Pajares, 2009), and students' physiological and emotional states such as stress and anxiety (Bandura, 1997). In college algebra, the exams in the course, on which much of a student's grade is based, could therefore have a large effect on students' mathematical self-efficacy beliefs, particularly if students score poorly on exams, causing the student to have less confidence in their abilities to succeed in mathematics in the future. Thus, one explanation for there being a statistically significant drop in students' self-efficacy in the traditional course as compared to the *Pathways* course might be students' interpretation of their actual performances on exams. In particular, in looking at the average score on the final exams within the two courses, the traditional course final exam average was 59.8%, as compared to the average on the *Pathways* final of 77.8%. With so many students scoring very low on the traditional exam, it is no wonder that students came away with lower self-efficacy, feeling that they may have essentially failed the final exam and done poorly in comparison to other students, even though the average was so low. For students at Brigham Young University, who are used to achieving high grades and scores, getting such a low score on a mathematics exam might cause students to feel that they are not very good at mathematics, and would therefore judge themselves to be less effective at using mathematics in the future.

While there is a good possibility that the traditional exams were actually more difficult than homework, or that they tested material never taught in the class, another possibility arises in examination of the homework in the traditional course. Although the online homework is an attempt to help students by showing them example problems and walking them through the solution step-by-step, this actually could be detrimental to their performance on exams, as Todd mentioned in one of his interviews:

Todd Although the "help me solve this" option on the homework is helpful in completing the problem, I rely too much on this and can't do it myself, and then it builds false confidence in my abilities, so when test time comes around, I no longer have the "help me solve this" option.

By focusing students solely on instrumental understanding (Skemp, 1978/2006), specifically walking the students through the solution of problems step-by-step on the online homework, students' mathematical confidence grows as they see themselves getting the correct answers. However, when they take the exam, they no longer have the online tools to get help, struggle greatly on problems they have not, or feel they have not, seen before, and get poor scores on the exam. This, then, plausibly causes their self-efficacy to drop, feeling that they cannot apply what they have learned in the course, and judging themselves to be unsuccessful in using mathematics in the future.

The results from the traditional course can be contrasted with those from the *Pathways* course by looking at the Self-Efficacy component questions which had gain scores that were statistically significant from zero in one sample t-tests, shown in Table 11.

Table 11

Self-Efficacy Questions which had Mean Gain Scores that were Significantly Different from Zero in the Pathways Course

Question	t	df	Significance	95% Confidence Interval	
				Lower	Upper
#2 I am sure I could do advanced work in mathematics	3.18	55	0.002	0.13	0.58
#17 I can get good grades in mathematics	-2.169	55	0.034	-0.52	-0.02

I make two observations about these results. First, students in the *Pathways* course may have been given a boost in their self-efficacy by receiving social persuasions (e.g. "I know you can do it") from others (Bandura, 1997). In the *Pathways* course, students are consistently told that participation in the *Pathways* course will help them to be more successful in their future mathematical studies. This is reinforced when they are told about the research done on the Pre-Calculus Assessment (PCA) (Carlson et al., 2010), and the fact that *Pathways* students consistently score higher on it than those in the traditional course. By being told that they will be more successful in their future mathematical studies by their instructors and by the PCA scores they receive, *Pathways* students might be socially persuaded that they will be successful in using mathematics in the future, and thus a stronger likelihood of increasing their self-efficacy, or at least their belief about being successful in advanced mathematics. Second, in contrast to the first observation, and a possible reason why there was no overall average effect to the Self-Efficacy component among *Pathways* students, students left the course with a slightly more negative view about their abilities to get good grades in mathematics. Once again, this may have been caused by students' views about the seemingly subjective nature of the grading within the *Pathways* course, discussed above within the Math as Sensible section. BYU students, who have very high GPA's coming into college, arguably have seen relatively high success in past mathematics courses. However, as the *Pathways* course focuses heavily on solution explanation and justification, students may struggle more with achieving high levels of success within the course than they had in past mathematics courses, causing a drop in their beliefs about their ability to get good grades within mathematics. In addition, many *Pathways* students voiced their frustration of the requirement for students to take the Mathematics Department final exam:

PS 1 I thought the math department [final] was terrible. We were not prepared to take it at all, and I think you could tell that from a lot of the scores.

Arguably, this exam was a high level of difficulty based on the average scores students achieved, and may have caused students to have less confidence in their abilities to get good grades in mathematics courses.

Still, it is telling that out of the three components of a mathematical disposition, the Self-Efficacy component was the only one which showed a statistically significant difference between the two courses. Further, the traditional college algebra course had an overall negative effect on students' self-efficacy beliefs, whereas the *Pathways* course had no overall effect. This suggests that *Pathways* could be considered a step in the right direction for improving students' self-efficacy beliefs within a college algebra course.

The Effects of College Algebra on Mathematical Dispositions Overall

As a last comparison of the effects of the college algebra courses on students' dispositions, I looked to see if the mean total gain score of all 38 questions from the Mathematical Disposition Survey (which could be considered a measure of a students' change in their overall disposition) was significantly different from zero in either of the courses. The results showed that only the traditional course had a statistically significant effect on students' overall dispositions, $t = -3.13,\ p = 0.006$, 95% CI [-15.62, -3.04], suggesting that the traditional course had an overall statistically significant negative effect on students' mathematical dispositions. By looking at the results above, this makes sense because the traditional college algebra course, on average, caused the Self-Efficacy and Perseverance components of a students' mathematical disposition to drop, whereas the *Pathways* course only had a negative effect on the Math as Sensible component.

Further Insights from the Interviews and Surveys on Students' Mathematical Dispositions

Diversity of student opinions about aspects of college algebra. Within the previous sections, I have mainly focused on average effects of the two courses on students' mathematical dispositions. However, another interesting result from my study was the huge amount of variation among students' gain scores on the three components of a mathematical disposition. For instance, consider the standard deviations within Table 12.

Table 12

Results from the MANOVA of the Mathematical Disposition Survey

Scale	Class	Mean Gain Score	Standard Deviation
Self-Efficacy	*Pathways*	-0.04	5.985
	Traditional	-3.50	5.294
Math as Sensible	*Pathways*	-1.93	6.051
	Traditional	-2.33	5.018
Perseverance	*Pathways*	-0.86	5.603
	Traditional	-3.56	5.447

These relatively high standard deviations give evidence of large variations in the effects of the college algebra courses on students' mathematical dispositions. Additionally, this large spread is also evidenced within individual students' gain scores. For example, students in the *Pathways* class ranged from having gain scores of -17 for Self-Efficacy, -13 for SMARTS, and -17 for Perseverance, up to +9 for Self-Efficacy, +12 for SMARTS, and +10 for Perseverance. This type of spread was also evident within the traditional college algebra course. Thus, how students' dispositions changed varied greatly within both courses.

Results from the interviews and surveys do give us some idea why there was so much variation in the effects on students' mathematical dispositions. On a large majority of aspects of the college algebra experience pointed out by students as notable, students opinions varied greatly. To give some idea of the diversity in student opinions, I present examples of the range of student vies on a variety of aspects within Figure 4.

Aspect of the Course	**Class**	**Positive View**	**Negative View**
Group Work	Pathways	*Finn* [One of my favorite aspects is] working with others to learn the content … teaching others and learning from others helps me establish things in my mind.	*Jon* It's clear sometimes that the class doesn't understand something, and so we get sent into the work book to just get the problems wrong. … The way the class was formatted, it almost deleted the option of being a visual learner. Almost everything was done by yourself or in groups and not shown to you.
Teacher and Teaching Assistant	Pathways (Speaking about the Same Teacher)	*Finn* My teacher! … Having a bright and positive teacher helps me be more excited to learn and do well.	*Luna* The way that my teacher teaches … I just didn't resonate with the teacher. Maybe I learn a different way than she teaches, but this class was really hard for me and frustrating.
	Traditional	*Todd* The TA … helps me be successful …	*Abe* [Also, one of my] least-favorite aspects of this course [is] the coldness of my TA.
Focus on Relational Understan-ding	Pathways	*PS 11* In many of my previous classes, we were told a rule of math, and were given no explanation of why it worked. But now that I know why certain rules work, it is easier for me to apply them, and math seems more doable. … This made me think that knowledge of why concepts work	*River* I don't like learning things purely conceptually. I don't like having information that might've been useful withheld because it was thought I couldn't combine it with application.

		really does help make a class more enjoyable.	
Homework	Traditional (Done online)	*Mark* … I really enjoy doing homework on the computer. It's so much easier and helpful!	*Todd* No, I do NOT. Because if there is just one little mistake you get the answer wrong, and if you can't find out why, you miss it and that doesn't help you learn anything. I like doing homework NOT on the computer and then coming into class and asking questions about the homework you got stuck on.
	Pathways	*Jon* [I like the homework]; it doesn't feel like a jump through the hoop exercise with 60 problems and you hand in a volume of papers of your work. It's direct, and we have the resources to help us, and we aren't graded on how we do … but if we do it.	*Kylie* Although I understand why, I wish the problems were closer to problems I am used to from high school or middle school. The more abstract aspects can be difficult for me.
Tests	Pathways	*Joe* I feel like the tests, homework, and other assignments have been manageable and I believe that we have been adequately prepared for them.	*River* I would only suggest a stronger correlation between the answers expected on homework and in class, and how they ought to be phrased on the test.
Pacing of the Course	Pathways	*Becky (P)* …the class moves at a comfortable pace, making it easy to keep up.	*PS 2* I thought the pace was a little slow, but that was based off of my own previous ability and knowledge. *PS 10* It seemed we moved too fast through material …
Textbook	Pathways	*PS 9* [One of my favorite aspects was] the book; it had good examples that were easy to understand.	*PS 13* I absolutely loathed the textbook/workbook. I found it to be useless as well as the furthest thing from clear.

Figure 4. Diversity of student opinions on aspects of the college algebra experience.

As seen in the students' responses throughout the results reported in Figure 4, there was huge diversity in students' views of many aspects of the course. Students were divided in their views about group work, the teacher, the instrumental or relational understanding (Skemp, 1978/2006) focus of the course, the textbook, the homework, and the tests given. With so much diversity in student opinions, it is not surprising that there was such a large spread in how students' dispositions towards mathematics changed as they took the different college algebra courses.

Expectations about mathematics courses. Another result from the interviews and surveys which might help to explain some of the diversity among the changes in students' mathematical dispositions are the expectations students held as they came into the course, and how these expectations led to students finding the course enjoyable or not. Quite possibly, the level of enjoyment that a student had in either of the two college algebra courses could have had an impact on students' dispositions towards mathematics. To show how students' expectations about the ways in which a mathematics course should run may have affected their mathematical dispositions, I will give four students as examples: Luna (*P*), Joe (*P*), Todd (*T*), and Abe (*T*).

Luna's recollection of past mathematics classes showed that her favorite aspect of a mathematics course is being taught a formula, how to use that formula, and then practicing with that formula:

Luna (P) My best math classes have always been algebra classes. I like working with formulas and numbers. However, whenever graphs and story problems come into the equation (no pun intended) that's when things start to get a little hard for me. … I have seen success when the teacher takes the time to go through and SHOW us how to do a problem, not just explain it with words.

However, promisingly, Luna caught on to the vision of the purpose of the *Pathways* course by the middle of the course, during the fourth interview, and seemed to desire to understand the mathematical concepts being taught:

Luna (P) The purpose of the class is to push me to be a critical thinker and to push myself to internalize the principals. I personally just want to see how all of the principles connect so that I can use math throughout my whole life … I think, at the beginning [of the course], I was just wanting to get through the class, but now I want to understand and use the skills I learn throughout my whole life. I want to actually internalize what I am learning.

Although it seemed Luna might end up really enjoying the course, and that her disposition towards mathematics would improve, Luna fell back into old habits, and later began to show a desire for a more traditional approach to teaching mathematics:

Luna (P) I don't think that I have mastered the principles behind the math problems that I have been doing. For now, I am just relying on remembering the formulas from the way I have seen them done in class.

...

I think what helped me the most [in past mathematics courses] was having the teacher go through the problems with us on the board instead of just putting a problem up and seeing what we think should happen. I just feel overwhelmed when that happens. I like when teachers just tell us how it is, and show us exactly what to do.

Thus, as Luna came to the end of the course, and was asked if she felt she would be successful in using mathematics in the future, she responded in the negative:

Luna (P) Not really to be honest. This class has been hard for me to understand. I just don't understand the principles the way that I would like to.

In the end, Luna's disposition had a zero gain score in the Perseverance component, a +4 gain score in the Math as Sensible component, and a -4 gain score in the self-efficacy component. Arguably, the harmful effect of *Pathways* to Luna's self-efficacy beliefs were likely the result of her expectations not being met about the way mathematics courses should be taught.

In contrast, Joe's recollection of past mathematics courses which he enjoyed were focused more on understanding concepts:

> *Joe (P)* My best math experience was probably geometry, actually. I felt like I really understood those concepts and I had an excellent teacher who explained and helped me understand why things worked/were the way that they are.
>
> …
>
> I like math when I feel like I understand it.

Even later in the class, during the fourth interview, Joe's desires for understanding the mathematical concepts still seemed very relevant:

> *Joe (P)* I hope to be able to pass this class with a decent grade, and I hope that I can retain the information that I am learning and actually apply it, instead of just getting through the course. I would love to understand, not just get by.

Later on in the course, Joe mentioned that the application of the concepts to real-world situations, group work, and helpfulness of the teaching assistants were all things that he really enjoyed about the course.

> *Joe (P)* I think these aspects have helped me know that regardless of whether I consider myself a "math person," I can still be effective in my math courses.

With the *Pathways* course focused heavily on a relational understanding (Skemp, 1978/2006) of mathematics, it seemed to satisfy Joe's desires to really understand the mathematical concepts being taught. Furthermore, the structure of the *Pathways* course might have helped Joe to feel that he would be more successful in using mathematics. Evidence of this can be found in the fact that Joe had a gain score of +3 in the Perseverance component, a gain score of +9 in the Self-Efficacy component, and no change in the Math as Sensible component.

Joe (P) I would recommend this class to anyone. I feel as though I have learned tactics and principles that I will use in my life and in future math classes.

Thus, one very plausible reason Joe's disposition towards mathematics improved was the alignment of aspects of the *Pathways* course to his desires for a mathematics class (e.g. learn with understanding, see the applicability of concepts to the real world, etc.).

In the traditional course, Todd's recollection of past mathematics courses showed his desire for understanding mathematical concepts, smaller class sizes, and time to work through and explore problems within the classroom:

Todd (T) I didn't really like math in high school because it was hard for me to understand. … I started actually enjoying math last semester when I took an intro to college algebra class at LDS Business College. I started liking it because the class size was small … and the teacher was patient and would not move on until you understood. He would take the time answering any question and would give you chances to practice on the board, and would walk us through problems we were not getting.

Later, during the third interview, Todd continued to show a desire to understand the mathematical concepts, but recognized that he was not gaining this understanding at times:

Todd (T) I think it is very important to understand what I am doing or else I won't get it. But sometimes I may not really understand why, and I just follow the steps I have learned and memorized.

The traditional college algebra class, and its focus on instrumental understanding (Skemp, 1978/2006), specifically memorizing steps and procedures to follow in order to get correct answers, did not serve Todd's desires well. Other elements of the course that did not satisfy

Todd's desires to really understand the mathematical concepts included the lectures, and the online homework:

Todd (T) I know the teacher is doing his best, but I just don't learn through math lectures. … I think the major reason people like me have a problem with math is because we don't learn through listening to lectures, or watching someone else work through the problem, but by going through problems, trial and error, with a guide to turn to if we get lost and explain specific questions.

…

Although the "help me solve this" option on the homework is helpful in completing the problem, I rely too much on this and can't do it myself, and then it builds false confidence in my abilities so when test time comes around, I no longer have the "help me solve this" option [and do poorly].

…

These things have really frustrated me during the semester and my view of math has gone down. I know, however, that when I understand math, it is more fun and enjoyable. I just feel like I spent the whole semester drowning and getting lost.

Again, as evidence of the effect of students' mathematical instruction desires on their college algebra experience, and their mathematical disposition, Todd had negative gain score sums for all three components of his mathematical disposition (-6 for Perseverance, -7 for Self-Efficacy, and -12 for Math as Sensible). Such a large drop in his dispositions may reasonably be at least somewhat attributed to the misalignment of the traditional college algebra course to his desires for a smaller class size, classes centered on student engagement rather than teacher lecture, and a class focused on helping students gain a true understanding of the mathematical concepts being taught.

Lastly, one of the strongest evidences for the effects of students' preconceived ideas about the elements that make up a successful mathematics course having an effect on their enjoyment of the course and their mathematical disposition was found with Abe. Abe had actually taken the *Pathways* course during a previous semester, but had not enjoyed it whatsoever. In contrast, Abe really seemed to enjoy the traditional course. In examining his responses to the E-mail interviews and the follow-up survey, there were two main reasons why Abe enjoyed the traditional college algebra course over the *Pathways* course. First, Abe much preferred the focus of the traditional homework on obtaining correct answers, and rather hated the focus on justification and explanation of solutions within the *Pathways* course:

Abe I had to write out all of my answers and the reasons behind those answers (even if I used the same methods throughout multiple problems) [within the homework of the Pathways course].

…

I like the way homework works in [the traditional] class. I like that it is online and easy to submit. I love that I can get tips on how to solve the questions and can work on it until I understand it and get it right.

Second, Abe felt that time was wasted within the *Pathways* classes, and not enough time was given to students to work on homework within the class; but, in the traditional course, the TA labs were mostly set aside for students to work on their homework:

Abe My instructor [in the *Pathways* course] taught only half of the given material during her lecture and then the TA taught the other portion of the material during lab. This made it so that I had to do all of my homework outside of the classroom whereas, in Math 110, I was allowed to work on most of my homework during my assigned lab hours. … I found more success in this Math 110 course because I took the time to do my homework and ask my TA questions. Because I had this option and resource I felt more enabled to succeed.

Arguably, Abe probably felt that the explorations of mathematical concepts, and the group work involved, within the *Pathways* course were unhelpful, and a waste of time; but, being given direct instructions on how to procedurally solve problems, and time to practice those procedures in the labs of the traditional course were very useful. In fact, when Abe talked about previous mathematics courses he had enjoyed, his description fit the traditional college algebra course's method of teaching very well:

Abe (T) My best mathematics class was sophomore year of high school. I appreciated this class because the teacher instructed us in a clean and consistent way. She had prepared slides that we would follow and work with. Then she would allow us time in class to work on our homework with each other and with her.

This alignment of the traditional college algebra course to Abe's desires for more direct instruction in mathematics, and the ability to work on homework in class, could be a large reason Abe's disposition towards mathematics showed improvement, with gain score sums of +9 in the Perseverance component, -1 in the Self-Efficacy component, and +2 in the Math as Sensible component.

With these four students as examples, it is clear that students' preconceived ideas about the aspects of a good or successful mathematics course might be having a large influence on their enjoyment of a particular college algebra course, and as a result, influence their dispositions towards mathematics.

Discussion

One of the most surprising results from this study was the average effect of the *Pathways* course on students' mathematical dispositions. On average, *Pathways* had no statistically significant effect on the Perseverance and Self-Efficacy components of students' mathematical dispositions, and actually had a negative effect on the Math as Sensible component. With one of the major goals of *Pathways* being a focus on student learning and success based on decades of research, aimed at increasing students' confidence in their mathematical and problem solving abilities (Carlson, 2013), it is startling that *Pathways* did not, on average, improve students' mathematical dispositions. However, this study may have given us some insight into why this was the case. First, certain concepts taught within college algebra, such as rational functions and imaginary numbers, may be particularly difficult for students to see as being personally useful. Second, and related to the first reason, the major goal of *Pathways* is not a terminal mathematics course, but rather a course specifically designed to better prepare students for calculus (Carlson et al., 2010; Huchendorf & Smith, 2013). Students who are taking college algebra as their last mathematics course (Small, 2006) have no need of a preparation of calculus or other higher level mathematics courses. Thus, when learning concepts, such as imaginary numbers, that are aimed at preparing students for later mathematical studies, these students' views of the uselessness of mathematics are only reinforced. Third, students who are accustomed to getting good grades in mathematics by simply finding correct answers may become very frustrated when their scores are effected by grading which is focused on assessing the soundness of arguments and justifications given for the solution. And fourth, certain aspects of the course, such as the Mathematics Department final exam and the online textbook, were mainly seen negatively. These problems with assessments and student resources may have made it much more difficult for students to see mathematics as sensible, useful, and worthwhile, and could have had a negative impact on students' self-efficacy beliefs, and beliefs about the importance of hard work and perseverance to mathematical success.

Another interesting result from this study concerns the large diversity of students' opinions about particular aspects of the two college algebra courses, as well as the large diversity in the effects of college algebra on students' mathematical dispositions. Within the *Pathways* course, this might partially be explained by research others have done looking at students' attitudes toward reform-based curriculum as they move from a more traditional mathematics curriculum. Reys et al. (1998) found that, within the 1st year of field testing various units from a reform-based curriculum within middle school mathematics classrooms, students' opinions and attitudes varied in regards to elements such as (1) working collaboratively in group work, (2) lessons that spanned several days, and (3) the content taught. Thus, students who moved from a more traditional curriculum in high school into the *Pathways* college algebra course, in experiencing new approaches to mathematical learning, would reasonably have varying opinions about group work and a focus on relational understanding (Skemp, 1978/2006). Furthermore, as reported in the study by Smith and Star (2007), students moving from a more traditional high-school class to a reform-based university course felt the most deeply affected by the change in how the mathematics class was conducted, and could thus be expected to have varying opinions of multiple aspects of the course. In fact, in a related study by Star, Smith, and Jansen (2008), they found great diversity and variation among students' opinions and views about both reform and traditional mathematics curriculums. Consider an implication these researchers gave:

> For teachers, researchers, and curriculum developers interested in the "impact" of current reforms, these results are a reminder that diversity in students' views of what is interesting, appropriate, challenging, helpful, and worthy in school mathematics are not unitary; they vary considerably.

The results of this present study corroborate these results reported by Star et al. (2008), and further suggest that this diversity of student views may help to explain the variations in the improvement or harm of college algebra on students' mathematical dispositions.

Lastly, one plausible reason for a student's opinions influencing the impact college algebra had on that student's mathematical disposition was the alignment or misalignment of the class to the student's expectations and preconceived ideas about the elements that constitute a successful and enjoyable mathematics course, and its impact on components of their disposition. These results are also supported by those of Smith and Star (2007), where they found that students dispositions that rose within a traditional mathematics course welcomed more challenging content, the absence of context, and more individual work; whereas students whose dispositions fell in that traditional course longed for a more reform-based classroom, and the features it entails, such as non-repetitive contextual (story) problems, group work, and understanding the mathematical concepts. Furthermore, results might suggest that students' views of the elements that make up a successful mathematics course, and beliefs about the usefulness of mathematics to their chosen careers or fields of study, are fairly well solidified by the time students reach college, thus making it harder to improve students' mathematical dispositions. This, in turn, could give some explanation as to why the two college algebra courses were not, on average, able to improve students' dispositions towards mathematics.

Chapter 5: Conclusion

Summary of the Study

This study was designed to add to our knowledge about factors that affect mathematical dispositions in a college algebra course by attempting to answer these two research questions:

1. What are the effects of college algebra classes (both a non-traditional, context-based course [*Pathways*], and a traditional course) on students' mathematical dispositions?
2. What aspects of the college algebra experience (*Pathways* and traditional) seem to or might have an effect on students' mathematical dispositions?

By better defining the components of a students' mathematical disposition by expounding on a productive disposition (National Research Council, 2001), and by surveying and interviewing students in two different college algebra classes (a traditional college algebra course, and the *Pathways* course that is more reform-based), I was able to compare and contrast the experiences of students in the two college algebra courses, and the resultant effects on students' mathematical dispositions.

On average, students in the *Pathways* course showed a drop in the Math as Sensible component of their mathematical dispositions, but no statistically significant difference in the Perseverance or Self-Efficacy components. In contrast, students in the traditional college algebra course, on average, showed no statistically significant difference in the Math as Sensible component of their mathematical dispositions, but did show a drop in the Self-Efficacy and Perseverance components, as well as a statistically significant drop in their overall dispositions. Results of the study further suggest that students come into college algebra with ideas already set in their minds about the components that make up a good mathematics course, and a large reason students' dispositions trend negatively or positively is due to the alignment of the course with their preconceived ideal math course.

Additionally, the results suggest possible aspects of college algebra that might affect students' mathematical dispositions, including: (1) the usefulness of specific content in the course to students' chosen careers or paths of study, as well as the amount that the content is contextualized in real-world applications, having an effect on students' abilities to see mathematics as rewarding, useful, and sensible (the Math as Sensible component); (2) the presence or absence of productive struggle (NCTM, 2014) having an effect on students' beliefs about the importance of hard work and determination to mathematical success (the Perseverance component); and (3) homework and exams, and students' judgments of their performance on them, having an effect on the confidence students have for being successful in using mathematics in the future (the Self-Efficacy component).

Contributions to the Mathematics Education Research Community

My study has contributed to the Mathematics Education research community in a number of ways. First, my thesis fleshes out and expounds upon the definition of a productive disposition originally defined by the National Research Council (2001). More specifically, I broke productive dispositions down into its three main components of self-efficacy, perseverance, and seeing mathematics as sensible, useful, and worthwhile, and more finely examined and defined what those components consist of, and how we might be able to help students develop a productive disposition, which was not done extensively by the National Research Council (2001). Furthermore, I explained how the three component spectrums (Math as Sensible; Self-Efficacy; Perseverance) can be used to define any students' disposition towards mathematics, whether it is productive or not. This is a step beyond research which has only talked about *positive* dispositions, or improving students' dispositions (National Research Council, 2001;

NCTM, 1989), and allows us to examine both positive and negative effects of mathematics courses on students' mathematical dispositions.

Second, using my expanded definition of mathematical dispositions, along with research that has been performed looking at self-efficacy, perseverance and problem solving, and the ability to see mathematics as useful and worthwhile, I was able to create the Mathematical Disposition Survey that was shown to have a high internal consistency, both in the pilot study and in the results of the actual study. This survey can now be used by others to examine the mathematical dispositions of students, as well as study how their dispositions might change over time.

Third, the results of my study have been able to better help us understand how college algebra can affect students' dispositions towards mathematics. In particular, the study has shown that college algebra classes, in general, seem to have an overall negative effect on students' mathematical dispositions. Still, based on the results of the Mathematical Disposition Survey and responses from students, a college algebra course such as the *Pathways* course that focuses on helping students gain a relational understanding (Skemp, 1978/2006), engaging students in productive struggle (NCTM, 2014), and specifically preparing students for future mathematics courses (Carlson et al., 2010; Huchendorf & Smith, 2013) might be a step in the right direction, as it may be less detrimental to students' dispositions than a traditional college algebra course.

Fourth, my study has given us some ideas about the differences in student experiences within two separate college algebra courses that may have had an influence on students' mathematical dispositions. Particularly, my study suggested that students' preconceptions of what a successful mathematical course entails may very well affect how the course is able to influence and change that student's mathematical disposition. Furthermore, I was able to pinpoint aspects of the course that might be having an effect on students' dispositions, such as the classroom structure (lecture vs. group work), the teacher and teaching methods (focus on procedural understanding vs. focus on conceptual understanding), and homework and assessments (how well the students feel the exams align with what is taught in the classroom and the homework). These aspects pointed out in the results give us a better idea of ways college algebra instructors and curriculum designers, as well as mathematics teachers in general, might be better able to help students develop a productive disposition towards mathematics.

Implications for Practice

Teachers can use the Mathematical Disposition Survey to get an overview of the dispositions of the students in their class at the beginning of a mathematics course to better plan lessons and activities that would be best suited for improving the dispositions of their students. Teachers can also use the Mathematical Disposition Survey multiple times during the course to evaluate the changes in students' dispositions, and see if their teaching and classroom environment is conducive to helping students develop a productive disposition towards mathematics (National Research Council, 2001). As teachers evaluate and assess where their students are, and where their students are heading in regards to their mathematical dispositions, they might be better able to change or focus their teaching practices on helping the dispositions of their students become more productive.

This study also corroborates ideas given by previous researchers, and gives further ideas, about specific ways teachers and instructors of mathematics can make efforts to improve their students' dispositions towards mathematics. First, teachers can embed the teaching of the specific mathematical concepts they teach within real-world contexts to help students see mathematics as useful and worthwhile (Brown et al., 1989). Second, teachers can give their students opportunities to engage in real problem solving and productive struggle (NCTM, 2014), giving them the chance to prove to themselves that they can be successful by pushing through obstacles and difficulties, and by reasoning with the mathematics themselves. Third, teachers might try to ensure that the summative assessments and exams that they give are not unreasonably difficult, do not test material that students never had the opportunity to learn, and accurately reflect the knowledge and understanding they should have gained within the classroom and displayed in the homework and assignments within the course. Fourth, and related to exams and assessment, the teacher could make sure that homework leads to understanding, and not simply on memorized procedures or step-by-step instructions given in the book or by an online program, making sure to help students who enjoy memorization ease into homework focused more on understanding, explanations, and justification; otherwise, students may become so reliant on being told exactly what to do, or being guided step by step, that when they are given an exam with no explicit instructions on how to solve problems, they are completely at a loss for even attempting the problems, causing their self-efficacy to plummet.

For college algebra in particular, there are also implications. First, the content of the college algebra course, as previous researchers have mentioned (Gordon, 2008; Small, 2006), may very well need to be called into question. Students in this study from both courses mentioned specific topics in the course (such as rational functions, imaginary numbers, and summations) as having little to no use in their lives or chosen careers. Thus, while certain mathematical topics are useful to mathematicians and specific STEM related fields, a good portion of the college algebra class teaches algebraic tricks and manipulations that are seen by many students as having no personal use whatsoever. Thus, college algebra instructors, and mathematics departments in general, might need to rethink college algebra, and focus on the mathematical concepts that will be useful to all students, regardless of their chosen field of study. Alternatively, the designers of college algebra courses might look for better ways of helping all students see concepts such as rational functions or summations as personally useful, perhaps looking for real-world applications that these concepts can be embedded within. By so doing, students' beliefs about mathematics being useful and worthwhile may increase. Second, few students mentioned that they really enjoy learning through lecture, thus alternative teaching styles should be explored, including engaging students in group work, or giving students opportunities in class to explore and delve into the mathematical concepts being taught. Third, designers of tests within college algebra need to be careful to not make the tests too difficult, making sure they are aligned with homework and what is taught in class, as overly difficult exams cause many students to feel as if they really do not understand mathematics, and have no hope of being successful at using mathematics in the future. Fourth, the textbooks used in college algebra, particularly those with the goal of helping students develop a rich understanding of the mathematical concepts taught in college algebra, need to be revised and improved with each iteration of the course. By changing and improving these aspects of the college algebra course, we may be able to stop college algebra from being so detrimental to their dispositions, and better help students' mathematical dispositions to become more productive.

Implications for Future Research

One of the biggest themes that arose from this study was the diversity in students' opinions about successful mathematics courses, and what makes those math classes successful. Arguably, students at the college level have gone through twelve years of schooling, and through their experience developed their view of the ideal mathematics class. When the college algebra course aligned with their preconceived ideal math class, they enjoyed college algebra, and their dispositions seemed more likely to improve. When the college algebra class did not align with their ideal math class, they struggled to enjoy the class at all, and their dispositions seemed more likely to drop. This, then, brings up the question: If students are so set in their views of mathematics and successful mathematics classrooms by the time they take college algebra, what time is the most crucial for developing students' views of mathematics into a productive disposition? Future research needs to examine how students' dispositions towards mathematics develop and change throughout their lives, and look to determine the most crucial times for parents and teachers to encourage the development of a productive disposition in their children and students. Previous research on the development of students' mathematical dispositions (Jansen, 2012; Smith & Star, 2007) may suggest that these crucial times are earlier in schooling rather than later.

In relation to the previous paragraph, another area of research could explore the following question: How do we improve students' mathematical dispositions if they are so set in their beliefs about mathematics and good mathematics courses by the time they enter college algebra? Perhaps teaching experiments within college algebra could be designed with a specific goal of improving students' dispositions towards mathematics, and explore ways of improving students' self-efficacy beliefs, beliefs about the importance of perseverance, and beliefs about the usefulness and sensibleness of mathematics.

Another implication for future research develops from the particular aspects of the course that were suggested by student responses as possible influences on the changing of their mathematical dispositions. While my study was able to point out aspects that students mentioned as being helpful or hurtful, and which I identify as possible influences on their mathematical dispositions, this study did not look at the extent to which these aspects effect students' mathematical dispositions. Thus, future research needs to take the particular aspects of college algebra that were frequently mentioned by students regarding their experiences with the course and study which aspects have the greatest influence on their dispositions towards mathematics. By doing so, college algebra instructors, as well as mathematics teachers in general, will be able to focus on the aspects of a mathematics course that have the greatest potential for helping students develop a productive disposition.

Limitations of this Study, and Further Research Implications

There are several limitations in this study to consider. First, this study had a very limited sample size to draw from, and thus any conclusions from the results of this study must be taken with caution. This was due to constraints on myself, the researcher, since I did not have the time or resources to get a larger sample size, as well as difficulties that arose in attempting to gather participants for the study. In order to remedy this limitation, I suggest that replications of this study be done to see if the commonalities found among students in my study hold true for larger samples of students. This future research might also capture aspects of the course that affect students' dispositions that I may have missed due to my limited sample size and time constraints.

Second, the participants in this study were all completely voluntary, and thus the results could very well be biased. However, this could not be avoided, as the human subjects approval required that participants in this study be asked on a voluntary basis. Still, I did my best to limit the biasing effects of a voluntary sample through (1) getting volunteers in both the *Pathways* and traditional courses, (2) making sure I had a spread of initial disposition levels among the E-mail interview participants in both classes, (3) using multiple data collection methods to try and get a broader view of the college algebra classes, and (4) working with as large a sample as my time, resources, and the students themselves would allow.

Third, and related to the first and second limitations, this study was only conducted at Brigham Young University. Because of this, future researchers should attempt similar studies at other universities to see if the commonalities found among students at BYU hold true among other populations of students. Furthermore, while the *Pathways* course at BYU was a good attempt to help improve college algebra by focusing on helping students gain a conceptual understanding, and see the usefulness of the mathematical concepts through contextualizing the concepts within real-world applications, there was a heavy influence from the Mathematics Department and their requirements for college algebra on the *Pathways* course. This, arguably, made it so the *Pathways* course could not quite function the way that it was intended, which plausibly confounded some of the results from the *Pathways* students. Future researchers need to see how completely redesigned college algebra courses, with little influence from traditional college algebra courses, effect students' mathematical dispositions, perhaps even looking at a *Pathways* course that is not required to meet traditional college algebra standards.

Fourth, as *Pathways* is specifically designed to help prepare students for calculus, it is a huge limitation that I did not ask students whether or not they intended to take calculus in the future. In hindsight, this would have been fairly simple to do, but this did not occur to me until after the data was all gathered. As such, future researchers looking at the effects *Pathways* has on students' mathematical dispositions should compare and contrast the dispositions of those who intend to go on to calculus with those who are taking college algebra as their very last mathematics course, specifically in their beliefs about the degree to which mathematics is sensible, useful, and worthwhile.

Fifth, no attempts were made within my study to examine or explore the extent to which the three components of students' mathematical dispositions correlate or influence one another.

To do this would have required a quite different study, and I did not have the time to explore this idea. Future research needs to examine how the three components of mathematical dispositions effect and influence one another, so we can gain an even better idea into ways we might be able to improve students' mathematical dispositions.

Sixth, this study did not look at correlations between a students' grades and their dispositions. While research has shown that self-efficacy is affected by students' perceptions of how well they performed (Schunk & Pajares, 2009), more research needs to be done to see how grades affect dispositions in general. More particularly, future researchers can use my expanded definition of dispositions in this study to see the effects of homework, exam, and final course grades on the three components of mathematical dispositions (Math as Sensible, Self-efficacy, and Perseverance). This will give teachers a better idea of which grades (homework, exams, and final grade) have the largest impact on students' dispositions, and possibly how they might be able to weight class grades to better help students develop a productive disposition.

Seventh, one semester is a relatively short time period to try and improve students' mathematical dispositions. Thus, a longer time period engaging in context-based, conceptual curriculums such as *Pathways* might be better able to improve a students' mathematical disposition. In fact, Bay, Beem, Reys, Papick, and Barnes (1999), as a follow-up to the Reys et al. (1998) study, found that students' attitudes towards mathematics greatly improved during the second year of engagement with a reform-based, conceptual curriculum. Thus, future research needs to explore if longer engagement in conceptual curriculums leads to more productive mathematical dispositions among students, and how long it would take to influence college students' mathematical dispositions for the better.

Eighth, and lastly, students in the traditional course may have had increases in their self-efficacy beliefs, and beliefs about perseverance leading to success, by engaging completely in a curriculum focused on instrumental understanding (Skemp, 1978/2006); Abe is an example of one such student. While this may have increased their dispositions on my Mathematical Disposition Survey, it might be argued that these increases were not really leading to a productive disposition, but were simply leading the student to feel more efficacious about their abilities to solve decontextualized mathematics problems. Such students might actually struggle to apply their knowledge and understanding to real-world applications or within future work. This suggests that future research should look at better distinguishing between truly productive self-efficacy beliefs, and beliefs about the importance of perseverance, and those beliefs which might develop more superficially.

In Conclusion

The purpose of this study was to better understand how students' experiences within mathematics courses, particularly college algebra, might effect and change a students' disposition towards mathematics. By looking at two different college algebra courses, a traditional course and the *Pathways* course, and the experiences of students within those courses, I found that students' dispositions, on average, were either not affected or negatively affected by college algebra, but that the *Pathways* course was less detrimental, and a step in the right direction towards improving students' dispositions towards mathematics through college algebra. Furthermore, I was able to discover specific aspects of the college algebra courses that seemed to have an effect on the students' dispositions, as well as demonstrate the diversity in students' opinions on many of these aspects. Lastly, I discovered that one plausible reason students' dispositions improved or dropped was the extent to which the college algebra course aligned with their beliefs about the features of a good or successful mathematics course. With these results, we can begin to work towards better improving college algebra, with the specific goal of uplifting students' mathematical dispositions as a main focus.

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Appendix A: Productive Mathematical Disposition Scale

The Self-Efficacy Component

Fennema-Sherman Confidence in Learning Mathematics Scale (Fennema & Sherman, 1976)

- \+ I am sure I could do advanced work in mathematics
- \+ I am sure that I can learn mathematics
- \+ I think I could handle more difficult mathematics
- \+ I can get good grades in mathematics
- – I don't think I could do advanced mathematics

Usher & Pajares (2009) Sources of self-efficacy in mathematics: A validation study

- \+ I can imagine myself working through challenging math problems successfully

Berger & Karabenick (2011) Motivation and students' use of learning strategies

- \+ I am confident I can learn the basic concepts taught in math

May (2009) Mathematics Self-Efficacy and Anxiety Questionnaire – Dissertation

- \+ I believe I can do well on a mathematics test
- \+ I believe I can complete all of the assignments in a mathematics course
- \+ I believe I will be able to use mathematics in my future career when needed
- \+ I believe I can learn well in a mathematics course
- \+ I believe I am the type of person who can do well in mathematics

The Math as Sensible Component

Fennema-Sherman Usefulness of Mathematics Scale (Fennema & Sherman, 1976)

- \+ I will need mathematics for my future work
- \+ I study mathematics because I know how useful it is
- \+ Mathematics is a worthwhile and necessary subject
- \+ I will use mathematics in many ways as an adult
- – Mathematics is of no relevance to my life
- – Mathematics will not be important to me in my life's work
- – I see mathematics as a subject I will rarely use in my daily life as an adult
- – Studying mathematics is a waste of time

Indiana Mathematics Belief Scales Developed By Kloosterman & Stage (1992)

- \+ Time used to investigate why a solution to a math problem works is time well spent
- \+ A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem
- \+ In addition to getting a right answer in mathematics, it is important to understand why the answer is correct
- – It's not important to understand why a mathematical procedure works as long as it gives a correct answer
- – Getting a right answer in math is more important than understanding why the answer works

– It doesn't really matter if you understand a math problem if you can get the right answer

The Perseverance Component

Indiana Mathematics Belief Scales Developed By Kloosterman & Stage (1992)

+ Math problems that take a long time don't bother me
+ I feel I can do math problems that take a long time to complete
+ I find I can do hard math problems if I just hang in there
– If I can't do a math problem in a few minutes, I probably can't do it at all
– If I can't solve a math problem quickly, I quit trying
– I'm not very good at solving math problems that take a while to figure out
+ By trying hard, one can become smarter in math
+ Working can improve one's ability in mathematics
+ I can get smarter in math by trying hard
+ Ability in math increases when one studies hard
+ Hard work can increase one's ability to do math
+ I can get smarter in math if I try hard

Appendix B: Mathematical Disposition Questionnaire (Pilot Study Version)

THIS SURVEY IS COMPLETELY VOLUNTARY. YOU ARE UNDER NO OBLIGATION TO COMPLETE IT.

Instructions: Read each item carefully and indicate the response (Strongly Disagree, Disagree, Uncertain, Agree, Strongly Agree) which best describes your feeling toward each statement. Do not spend too long on any given item. Mark your response on the bubble sheet provided using a #2 Pencil. For data organization purposes, please also fill out the "IDENTIFICATION" box on the bubble sheet using your student ID number. Your answers to this questionnaire will remain anonymous.

Item #		Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
1	I study mathematics because I know how useful it is	1	2	3	4	5
2	I am sure I could do advanced work in mathematics	1	2	3	4	5
3	I'm not very good at solving math problems that take a while to figure out	1	2	3	4	5
4	I am confident I can learn the basic concepts taught in math	1	2	3	4	5
5	I see mathematics as a subject I will rarely use in my daily life as an adult	1	2	3	4	5
6	I can get smarter in math by trying hard	1	2	3	4	5
7	Getting a right answer in math is more important than understanding why the answer works	1	2	3	4	5
8	I believe I can learn well in a mathematics course	1	2	3	4	5
9	I will use mathematics in many ways as an adult	1	2	3	4	5
10	If I can't do a math problem in a few minutes, I probably can't do it at all	1	2	3	4	5
11	I believe I will be able to use mathematics in my future career when needed	1	2	3	4	5
12	In addition to getting a right answer in mathematics, it is important to understand why the answer is correct	1	2	3	4	5
13	By trying hard, one can become smarter in math	1	2	3	4	5
14	Mathematics is a worthwhile and necessary subject	1	2	3	4	5
15	I believe I can complete all of the assignments in a mathematics course	1	2	3	4	5
16	Hard work can increase one's ability to do math	1	2	3	4	5

Item #		Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
17	I can get good grades in mathematics	1	2	3	4	5
18	It's not important to understand why a mathematical procedure works as long as it gives a correct answer	1	2	3	4	5
19	I think I could handle more difficult mathematics	1	2	3	4	5
20	Ability in math increases when one studies hard	1	2	3	4	5
21	Studying mathematics is a waste of time	1	2	3	4	5
22	Working can improve one's ability in mathematics	1	2	3	4	5
23	I believe I can do well on a mathematics test	1	2	3	4	5
24	I will need mathematics for my future work	1	2	3	4	5
25	I am sure that I can learn mathematics	1	2	3	4	5
26	Math problems that take a long time don't bother me	1	2	3	4	5
27	I believe I am the type of person who can do well in mathematics	1	2	3	4	5
28	It doesn't really matter if you understand a math problem if you can get the right answer	1	2	3	4	5
29	I find I can do hard math problems if I just hang in there	1	2	3	4	5
30	Mathematics will not be important to me in my life's work	1	2	3	4	5
31	I can get smarter in math if I try hard	1	2	3	4	5
32	I don't think I could do advanced mathematics	1	2	3	4	5
33	I feel I can do math problems that take a long time to complete	1	2	3	4	5
34	Mathematics is of no relevance to my life	1	2	3	4	5
35	I can imagine myself working through challenging math problems successfully	1	2	3	4	5
36	Time used to investigate why a solution to a math problem works is time well spent	1	2	3	4	5
37	If I can't solve a math problem quickly, I quit trying	1	2	3	4	5
38	A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem	1	2	3	4	5

Appendix C: Mathematical Disposition Questionnaire

THIS SURVEY IS COMPLETELY VOLUNTARY. YOU ARE UNDER NO OBLIGATION TO COMPLETE IT.

Please fill out the following information. These will be used for data organization and analysis purposes only. Your information will be kept secure, and all identifiers will be removed from reports, and from your responses at the end of this study.

By participating in this survey, you are giving your implied consent to be a participant in this study. You are also giving your consent to be contacted at the end of this course to take the survey again. You may withdraw from the study at any time.

E-Mail ____

Student ID # ________________________ Major ___________________ Gender: M F

Have you taken College Algebra before? YES NO Age: 18-25 26-30 31+

Instructions: Read each item carefully and indicate the response (Strongly Disagree, Disagree, Uncertain, Agree, Strongly Agree) which best describes your feeling toward each statement. Do not spend too long on any given item. Circle your responses below.

Item #		Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
1	I study mathematics because I know how useful it is	1	2	3	4	5
2	I am sure I could do advanced work in mathematics	1	2	3	4	5
3	I'm not very good at solving math problems that take a while to figure out	1	2	3	4	5
4	I am confident I can learn the basic concepts taught in math	1	2	3	4	5
5	I see mathematics as a subject I will rarely use in my daily life as an adult	1	2	3	4	5
Item #		Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
6	I can get smarter in math by trying hard	1	2	3	4	5
7	Getting a right answer in math is more important than understanding why the answer works	1	2	3	4	5
8	I believe I can learn well in a mathematics course	1	2	3	4	5
9	I will use mathematics in many ways as an adult	1	2	3	4	5
10	If I can't do a math problem in a few minutes, I probably can't do it at all	1	2	3	4	5
Item #		Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
11	I believe I will be able to use mathematics in my future career when needed	1	2	3	4	5
12	In addition to getting a right answer in mathematics, it is important to understand why the answer is correct	1	2	3	4	5
13	By trying hard, one can become smarter in math	1	2	3	4	5

Item #		Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
14	Mathematics is a worthwhile and necessary subject	1	2	3	4	5
15	I believe I can complete all of the assignments in a mathematics course	1	2	3	4	5
16	Hard work can increase one's ability to do math	1	2	3	4	5
17	I can get good grades in mathematics	1	2	3	4	5
18	It's not important to understand why a mathematical procedure works as long as it gives a correct answer	1	2	3	4	5
Item #		Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
19	I think I could handle more difficult mathematics	1	2	3	4	5
20	Ability in math increases when one studies hard	1	2	3	4	5
21	Studying mathematics is a waste of time	1	2	3	4	5
22	Working hard in a math class can improve one's ability in mathematics	1	2	3	4	5
23	I believe I can do well on a mathematics test	1	2	3	4	5
Item #		Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
24	I will need mathematics for my future work	1	2	3	4	5
25	I am sure that I can learn mathematics	1	2	3	4	5
26	Math problems that take a long time don't bother me	1	2	3	4	5
27	I believe I am the type of person who can do well in mathematics	1	2	3	4	5
28	It doesn't really matter if you understand a math problem if you can get the right answer	1	2	3	4	5
Item #		Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
29	I find I can do hard math problems if I just hang in there	1	2	3	4	5
30	Mathematics will not be important to me in my life's work	1	2	3	4	5
31	I can get smarter in math if I try hard	1	2	3	4	5
32	I don't think I could do advanced mathematics	1	2	3	4	5
33	I feel I can do math problems that take a long time to complete	1	2	3	4	5
Item #		Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
34	Mathematics is of no relevance to my life	1	2	3	4	5
35	I can imagine myself working through challenging math problems successfully	1	2	3	4	5
36	Time used to investigate why a solution to a math problem works is time well spent	1	2	3	4	5
37	If I can't solve a math problem quickly, I quit trying	1	2	3	4	5
38	A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem	1	2	3	4	5

Appendix D: E-mail Interview Questions

First E-mail interview questions

1. Tell me about your best and worst mathematics classes, and why they were that way.

2. Tell me whether or not you like math, and why it is you feel that way.

3. Have you always felt the same about mathematics? If not, how has it changed?

4. What does it take to be successful in a math class?

Second E-mail interview questions

1. Do you think you will be successful in this College Algebra course and, why or why not?

2. Why are you taking Math 110?

3. Why do you think your major requires it, or that it is a prerequisite for a class you need in your major?

4. Do you think you will use it?

5. What do you think you're getting out of the class?

Third E-mail interview questions

1. When you work on your homework, where do you like to do it, and who do you like to do it with (alone or with others)?

2. What happens when you run into a problem that you have trouble with? Do you work on it for a while, or just skip it?

3. Do you think it is important to understand what you are doing when you work on your homework?

4. Do you like the way the homework works in your class, and why or why not?

Fourth E-mail interview questions

1. From the teacher's perspective, what is the purpose of this class?

2. From the curriculum developer's [writer of the text book] perspective, what is the purpose of this class?

3. From your perspective [and please be as honest as possible], what do you think the purpose of this class is, and what do you personally want to get out of this class?

4. In regards to what you want to get out of this class, have these wants and desires changed since the beginning of the semester?

5. What do you think the purpose of mathematics is?

<u>Fifth E-mail interview questions</u>

1. What aspects of the class are helping you be successful in this class?

2. What aspects about you are helping you be successful in this class?

3. Is there anything about you or this class that you feel is hindering your success?

4. If you were given a mathematics problem related to the topics you have studied, but different from homework problems you have seen so far, do you think you would be successful in solving it? If so, why do you feel that way?

<u>Sixth E-mail interview questions</u>

1. Thinking back on past mathematics courses, what things best helped you understand mathematical concepts?

2. Thinking about this college algebra class, what things about it are best helping you understand the mathematical concepts you are learning?

3. When you say “I understand [a certain topic from this college algebra class] really well,” what would you mean by “understand?”

4. What would you define as being “successful” in a mathematics course? Name at least three aspects that “success” in a mathematics course encompasses.

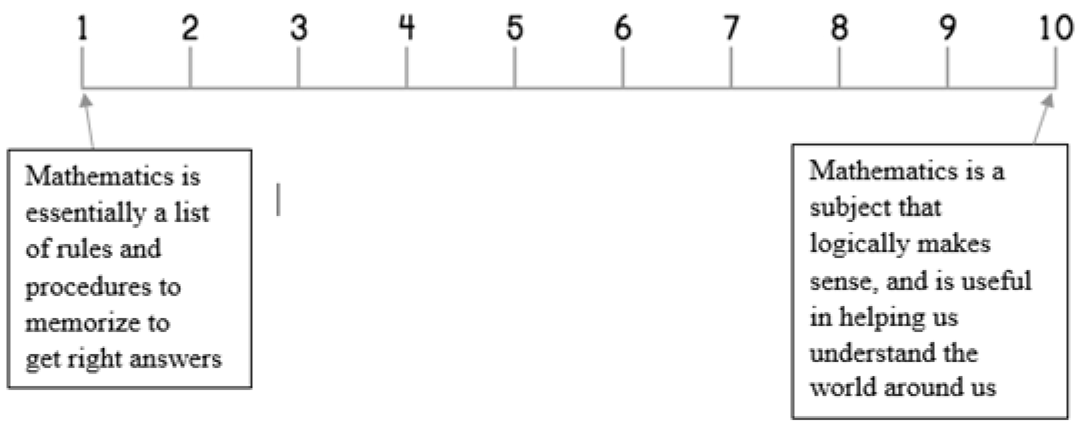
Seventh E-mail interview questions

1. How much do you feel the mathematics you have learned in this class will be personally useful to you?

2. If possible, can you list two to three mathematical concepts you have learned this semester that you see as being personally useful, and why they are so?

3. Do you think mathematics in general is something worthwhile for you? Why?

4. Locate how you feel on the following spectrum, and explain why you feel that way:



Eighth E-mail interview questions

1. Can you name your three **favorite** aspects of this college algebra course?

2. How have these aspects changed your personal view of mathematics?

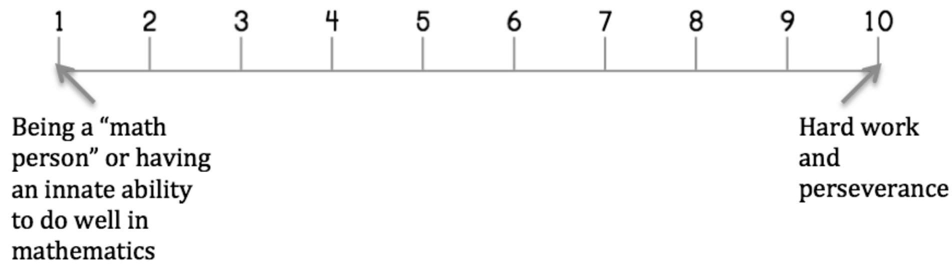
3. Can you name your three **least favorite** aspects of this college algebra course?

4. How have these aspects changed your personal view of mathematics?

Ninth E-mail interview questions

1. Locate yourself on the following spectrum, and explain why you feel that way:

The most important factor that leads to success in mathematics is



2. Do you feel like you will be successful in using mathematics in the future (either in your work or in future classes), and why do you feel that way?

3. Would you recommend this class to other students, and why or why not?

4. If you could, what would you change about the course, and why would you change it?

5. Do you have any other comments about the course, or your views about mathematics in general that you would be willing to share?

Appendix E: Screen Shots of College Algebra Experiences Survey

BYU

BRIGHAM YOUNG

UNIVERSITY

Implied Consent—College Algebra Experiences Survey

My name is Kevin Watson, I am a graduate student at Brigham Young University and I am conducting this research under the supervision of Professor Steve Williams, of the Mathematics Education Department. You are being invited to participate in this research study of "Examining the Effects of College Algebra on Students' Mathematical Dispositions." I am interested in finding out about the aspects of a college algebra course that may have an effect upon students' dispositions towards mathematics.

Your participation in this study will require the completion of the following open-ended survey. This should take approximately 20 minutes of your time. Your participation will be anonymous and you will not be contacted again in the future (unless you indicate willingness to be contacted about a future interview). You will not be paid for being in this study. This survey involves minimal risk to you. The benefits, however, may impact society by helping increase knowledge about how college algebra affects student dispositions towards mathematics, and ways in which we can improve college algebra to better help students' develop a productive disposition towards mathematics.

You do not have to be in this study if you do not want to be. You do not have to answer any question that you do not want to answer for any reason. We will be happy to answer any questions you have about this study. If you have further questions about this project or if you have a research-related problem you may contact me, Kevin Watson, at watsonkevin88@gmail.com; (801) 422-3711; 145 TMCB, Brigham Young University, Provo, UT 84602; or my advisor, Steve Williams at 193B TMCB, Brigham Young University, Provo, UT 84602; williams@mathed.byu.edu; (801) 422-7784.

If you have any questions about your rights as a research participant you may contact the IRB Administrator at A-285 ASB, Brigham Young University, Provo, UT 84602; irb@byu.edu; (801) 422-1461. The IRB is a group of people who review research studies to protect the rights and welfare of research participants.

The completion of this survey implies your consent to participate. If you choose to participate, please click the button below, and answer the questions to the best of your ability. Thank you!

Survey Completion

0% 100%

>>

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Did you use the *Pathways College Algebra* textbook by Carlson?

- Yes
- No

When you say "I understand [a certain topic from this college algebra class] really well," what would you mean by "understand?"

What would you define as being "successful" in a mathematics course? Name at least three aspects that "success" in a mathematics course encompasses.

Can you name your three **favorite** aspects of this college algebra course? How have these aspects changed your personal view of mathematics?

Can you name your three **least favorite** aspects of this college algebra course? How have these aspects changed your personal view of mathematics?

Survey Completion

0% 100%

>>

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Locate how you feel on the following spectrum:

	0	1	2	3	4	5	6	7	8	9	10
	Mathematics is essentially a list of rules and procedures to memorize to get right answers										Mathematics is a subject that logically makes sense and is useful in helping us understand the world around us
What is Mathematics?											

Why do you feel this way about mathematics?

Locate how you feel on the following spectrum:

	0	1	2	3	4	5	6	7	8	9	10
	Being a "Math person" or having an innate ability to do well in mathematics										Hard work and perseverence
The most important factor that leads to success in mathematics is:											

Why do you feel this way about being successful in mathematics?

Survey Completion
0% 100%

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Do you feel like you will be successful in using mathematics in the future (either in your work or in future classes), and why do you feel that way?

Would you be willing to be contacted for a 30-minute interview in the near future about your college algebra experience?

- Yes (Please Enter you E-mail address in the box provided)
- No

Survey Completion

0% 100%

>>

Survey Powered By Qualtrics

Appendix F: Follow-Up Surveys

Questions given to Abe:

1. What are some things that made Pathways unenjoyable for you? (Name at least three)

2. What are the biggest differences between the Pathways course and this last Math 110 course that you took? (Name at least three)

3. What about this latest Math 110 course made it possible for you to succeed and enjoy the mathematics you were learning?

4. Why do you feel more confident about mathematics? Are there specific aspects of this last Math 110 class that have helped you become more confident, and if so, what are they?

5. Do you think mathematics will be useful to you in the future? Why or why not? And, were there aspects of the course that helped you see mathematics as useful? If so, what were they?

Questions given to Nadine:

1. What are some things that made your class enjoyable? (Name at least 3 if possible)

2. What are some things that made your class unenjoyable? (Name at least 3 if possible)

3. Do you feel more confident about mathematics? What specific aspects of the college algebra course made you feel this way?

4. Do you think mathematics is useful to you? What specific aspects of the college algebra course helped you feel this way?

5. What is more important to succeed in mathematics: talent or hard work? What specific aspects of the college algebra course helped you feel this way?

6. Did you change majors based upon your experience in College Algebra? Why or why not?